Summer 2016
Lesson Plan Project for Discrete Math

Geometry lessons, 7th and 8th grade lessons and Algebra lesson

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Discrete Math Table of Contents

What is Discrete Mathematics?

It is an important branch used in business and industry. Discrete math is mixed into many of our current math standards in Minnesota. Some of the typical high school topics we see it in discrete math are vertex-edge graphs, matrices, problem solving, analyzing step by step procedures, voting methods, cryptography, finding the best solution from multiple solution paths, coding theory and game theory. Discrete math contracts with continuous math in calculus.

Overview

In our discrete math project we started by looking at the attributes and properties that different polygons possess. In our Building Polygons lesson we looked at side lengths and used poly strips to see the relationships between side lengths in triangles and quadrilaterals. We then looked at the characteristics and patterns of angles, both interior and exterior, in triangles and other polygons. In Shape Design we continued with this theme and looked for different ways to group polygons based on their attributes. We used one lesson as a way for student to learn the Pythagorean Theorem before we switched gears and got into some counting (permutation, combination) ideas with the Flag lesson. Our final lesson involved weighted vertex edge graphs and some of the cities near our hometowns.

MN --- Standards used:

9.3.3.2 Know and apply properties of angles, including **corresponding**, **exterior**, **interior**, **vertical**, **complementary** and **supplementary angles**, to solve problems and logically justify results.

9.3.3.3 Know and apply properties of **equilateral**, **isosceles** and **scalene triangles** to solve problems and logically justify results.
For example: Use the triangle inequality to prove that the perimeter of a quadrilateral is larger than the sum of the lengths of its diagonals.

9.3.3.7 Use properties of **polygons**—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results.
For example: Recognize that a rectangle is a special case of a trapezoid.
Another example: Give a concise and clear definition of a kite.

9.3.4.6 Use numeric, graphic and symbolic representations of **transformations** in two dimensions, such as **reflections**, **translations**, **scale changes** and **rotations** about the origin by multiples of 90°, to solve problems involving figures on a coordinate grid. *For example*: If the point (3,-2) is rotated 90° counterclockwise about the origin, it becomes the point (2, 3).

___ 7.3.1.1 - Demonstrate an understanding of the proportional relationship between the

diameter and circumference of a circle and that the unit rate (constant of proportionality) is . Calculate the circumference and area of circles and sectors of circles to solve problems in various contexts.

___ 7.3.1.2 - Calculate the volume and surface area of cylinders and justify the formulas used.

___ 7.3.2.1 - Describe the properties of similarity, compare geometric figures for similarity, and

determine scale factors.

___ 7.3.2.2 - Apply scale factors, length ratios and area ratios to determine side lengths and

areas of similar geometric figures.

___ 7.3.2.3 - Use proportions and ratios to solve problems involving scale drawings and

conversions of measurement units.

___ 7.3.2.4 - Graph and describe translations and reflections of figures on a coordinate grid and

determine the coordinates of the vertices of the figure after the transformation.

??????
7th grade easy to understand:

0. Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Sample MCA Grade 11 Question

1. Roads connecting the towns of Oceanside, River City, and Lake View form a triangle. The distance from River City to Oceanside is 38 kilometers. The distance from River City to Lake View is 26 kilometers. What is the smallest possible whole number of kilometers from Lake View to Oceanside?

Triangle ABC is a right triangle. The shortest side is 7 m long and the longest side is 10 m long. What is the approximate length of the third side?

A. 3 m

B. 7 m

C. 12 m

D. 17 m

1 7 13 19 ...

8. Mr. Davis wrote the above arithmetic progression on the board. What are the next two numbers in this progression?

A. 21 27
B. 23 29
C. 25 31
D. 27 33

Name______________________

Discrete Math Pretest

1. Find the sum of the interior angles in a convex 16-gon.

2. Find the sum of the exterior angles in a convex 20-gon.

3. Find the measure of one interior angle of a regular 30-gon.

4. Find the measure of one exterior angle of a regular decagon.

5.Follow these directions for 5a and 5b

• If possible, build a triangle with the given set of side lengths. Sketch your triangle.

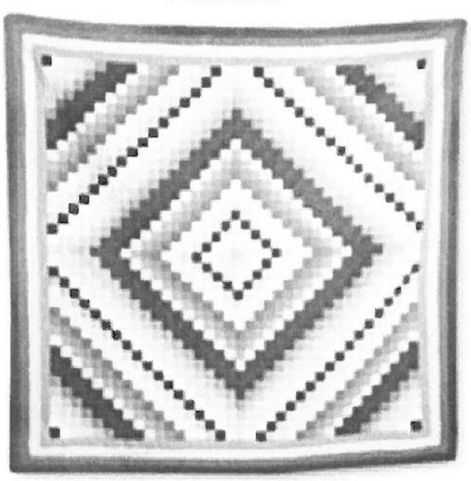
• Tell whether your triangle is the only one that is possible. Explain.

• If a triangle is not possible, explain why.

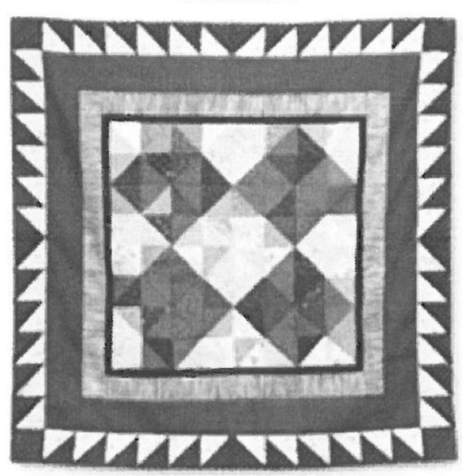
5a. Side lengths of 5, 5, and 3   5b. Side lengths of 8, 8, and 8

6.

Pattern A



Pattern B



6. Name some of the polygons in each quilt pattern.

7. Describe the symmetries of each quilt pattern.

8. Draw and label the polygon with the following properties.

<ABC = 90, <BCA = 30, and side BC = 3 in.

Flag Trademarks Lesson 1 (with 2 advitiy for the day)

This problem's focus is to give students a concrete context that helps them distinguish between the multiplication and Addition principle of counting.

Goals:

Represent, analyze, and solve counting problem that do or do not involve ordering and that do and do not involve repetition.

Understanding and apply the addition and multiplication principles of counting, and represent these principles with algebra, including factorial notation.

Before review tree diagrams

Materials:

- Copy of the "Flag Trademarks" activity sheet
- Scissors
- Glue or tape
- Poster board (something to display flag cut-outs)
- Markers
- A bucket containing four colored blocks: one red, one green, one blue, and one yellow
- Three copies of the "Flag Cutouts" template (one red, white, blue paper) for each pair of students

Activity 1: Possible Outcomes with Replacement and Without Replacement

Launch: Replacement

Teacher has has a bucket with one red, one green, one blue and one yellow block in it. How many different outcome can we pull from the bucket?

1) Have students work in partners to draw a tree diagram to prove that all possible outcomes were listed.

2) Point out that the tree diagram shows the multiplication principle if we read it across: 4x4=16 possible outcomes. If read just the outcomes, we can use the addition principle: 4+4+4+4=16 possible outcomes.

3) Ask students to use the tree diagram to find the answers to the multiple situations: • how many outcomes produce exactly one red blocks? (Three branches

of red + one branch from each of the others produces 3+1+1+1=6 of the 16 outcomes produce one red blocks.) • how many outcomes produce first a red block, then another color? (3) • how many outcomes produce one green ball and one red block? (2) 16 • how many outcomes produce first a green block then a red block?

Launch: with - out replacment

Ask students, "how would the tree diagram be different if we did not allow replacement?" Brainstorm ideas.

1) Have students work in partners to draw a tree diagram to prove that all possible outcomes were listed. Ask students to verbalize how this new tree diagram is different from the previous tree diagram. (This tree diagram shows shows the multiplication principle if we read it across: 4x3=12 possible outcomes. If read just the outcomes, we can use the addition principle: 3+3+3+3=12 possible outcomes.)
2) Ask students to use the tree diagram to compare the outcomes from this tree diagram to the previous one. Which questions have the same answers? Which ones have different answers?

Activity 2:

Flag Trademarks Investigation

Launch:

A luggage company is considering two rectangular flag patterns with three possible colors. The winning design will be chosen to be used as a trademark for the company. How many outcomes are possible? Does one of the patterns have more possible outcomes than the other?

Explore

1) Before starting, discuss when the models are the same, and when they are different. For example, if a half-turn produces the same model, then it is not considered another outcome.

2) Pair students in partners or groups of three. Distribute the "Flag Cutout" sheets to each group. Students will cut the models to create a poster that displays the possible outcomes for the patterns A and B.

3) Ask students to prove they have a picture of all possible outcomes with a tree diagram (some of the branches on the tree diagram will have to be eliminated to avoid duplication of patterns).

4) Encourage students to explore what counting principle might help calculate the possible outcomes for each pattern (pattern B has 6 outcomes, pattern A has 9).

Share:

Have each group put up posters of their flags with their solution, Then different members to share why?

Summarize: from intro:

After students are comfortable with each tree diagram, discuss the counting principle for each diagram. Ask students to verbalize why the counting is different for each tree diagram, even though they both utilize the same bucket of blocks.

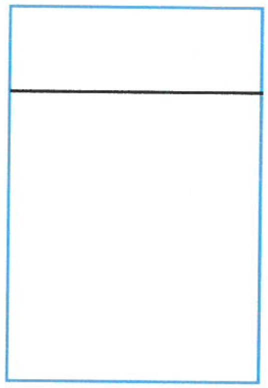
Summarize:

Students should conclude that forming tree diagrams is helpful for visualizing the counting principles, but tree diagrams become cumbersome when the number of outcomes becomes too large. Students should be able to apply the counting principle directly to problems like this. Explain that using the counting formula is also a form of what we call "Permutations."

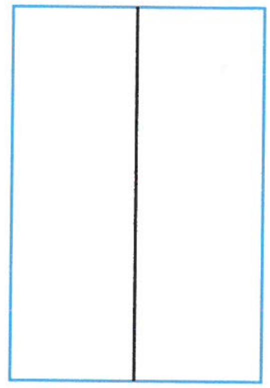
Flag Trademarks

Name ______________________________

A luggage company is considering two rectangular flag patterns, pattern A and pattern B, for use as a trademark. The company plans to use the winning pattern on one side of a certain brand of luggage in the orientation shown below.



Pattern A



Pattern B

Note that pattern A looks different when it is rotated 180° about its center, though pattern B looks just the same when rotated 180° about its center. Each pattern requires two pieces of cloth.

1. Suppose that the company plans to use red, white, and blue cloth for prototypes of the flag trademarks. You will use red, white, and blue construction paper to cut out the shapes in patterns A and B so that you can create models of the trademarks. You must display all the different possibilities for patterns A and B on a poster. You are trying to save time and space, so you want to be sure that you produce the fewest cutout models that can represent all possible patterns. Use the construction paper to make the flags, and mount them on poster board or paper. But first, count the number of possibilities by following the steps below (write your explanations on a separate sheet of paper).

   a. If the two pieces of cloth can be the same color, how many different pattern A models do you need to make? Explain how you determined your total, and display each pattern on the poster.

   b. If the two pieces of cloth can be the same color, how many different pattern B models do you need to make? Explain how you determined your total, and display each pattern on the poster.

   c. If the two pieces of cloth must be different colors, how many different pattern A models do you need to make? Explain how you determined your total, and display each pattern on the poster board.

   d. If the two pieces of cloth must be different colors, how many different pattern B models do you need to make? Explain how you determined your total, and display each pattern on the poster board.

Flag Trademarks (continued)

Name ______________________________

2. Suppose that the company plans to use red, white, blue, and green cloth for prototypes of the flag trademarks. Construct a tree diagram that shows all possible different color pairs. Use the diagram to answer the following questions:

   a. If the two pieces of cloth can be the same color, how many different pattern A models are possible? Explain how you determined your total.

   b. If the two pieces of cloth can be the same color, how many different pattern B models are possible? Explain how you determined your total.

   c. If the two pieces of cloth must be different colors, how many different pattern A models are possible? Explain how you determined your total.

   d. If the two pieces of cloth must be different colors, how many different pattern B models are possible? Explain how you determined your total.

3. Suppose that the company plans to use red, white, blue, green, and yellow cloth for prototypes of the flag trademarks. Use counting principles to answer the following questions:

   a. If the two pieces of cloth can be the same color, how many different pattern A models are possible? Explain how you determined your total.

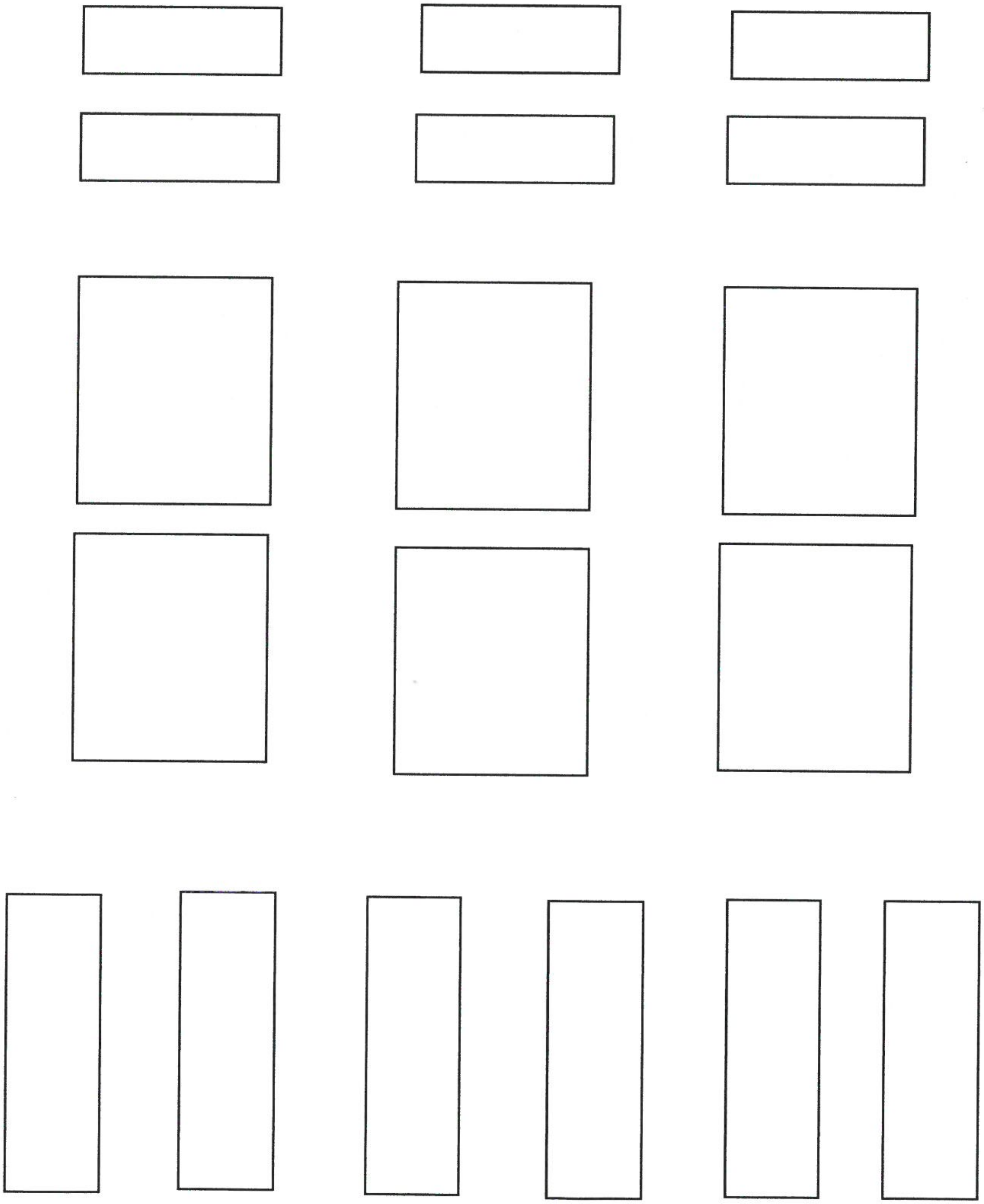
   b. If the two pieces of cloth can be the same color, how many different pattern B models are possible? Explain how you determined your total.

   c. If the two pieces of cloth must be different colors, how many different pattern A models are possible? Explain how you determined your total.

   d. If the two pieces of cloth must be different colors, how many different pattern B models are possible? Explain how you determined your total.

4. Design your own flag trademark pattern that requires three different colors. If you can choose from a selection of eight colors, how many different models are possible for your pattern? Explain how you determined your total.

Templates for Flag Cutouts



Shapes and Designs

Goal of this unit.

Properties of Polygons: Understand the properties of polygons that affect their shape.

- Explore the ways that polygons are sorted into families according to the number and length of their sides and the size of their angles
- Explore the patterns among interior and exterior angles of a polygon
- Explore the patterns among side lengths in a polygon
- Investigate the symmetries of a shape-rotation or reflection
- Determine which polygons fit together to cover a flat surface and why
- Reason about and solve problems involving various polygons

Relationships Among Angles: Understand special relationships among angles.

- Investigate techniques for estimating and measuring angles
- Use tools to sketch angles
- Reason about the properties of angles formed by parallel lines and transversals
- Use information about supplementary, complementary, vertical, and adjacent angles in a shape to solve for an unknown angle in a multi-step problem

Constructing Polygons: Understand the properties needed to construct polygons.

- Draw or sketch polygons with given conditions by using various tools and techniques such as freehand, use of a ruler and protractor, and use of technology
- Determine what conditions will produce a unique polygon, more than one polygon, or no polygon, particularly triangles and quadrilaterals
- Recognize the special properties of polygons such as angle sum, side-length relationships and symmetry, that make them useful in building, design, and nature
- Solve problems that involve properties of shapes

Investigation 1 The Family of polygons

1.1 Sorting and Sketching Polygons

Lesson 1--for 7th grade-1day

This problem's focus is on extending students' knowledge of polygons and showing how groups polygons by important common properties.
For students to understand the organization of polygons, they sort polygons into groups according to categories that they think make sense. We can assess prior knowledge of common shapes and how side lengths and angles determine the shape of a polygon.

Launch: **What properties do all polygons share? What properties do some sub-groups of polygons share?**

Display the example of polygon and nonpolygon shapes. Ask students to study the examples of polygons and nonpolygons.
Suggested Questions

- How would you describe differences between polygons and nonpolygons?
- What test would you use to decide if a figure is or is not a polygon?
- What familiar objects have shapes like the polygon examples?
- What other shapes are examples of polygons and nonpolygons?

Explore:

As we move about the room, look for more clues about students' knowledge of shapes. Ask groups about special polygons. Look for opportunities to ask questions about symmetry, particularly reflection.
Questions

- Can you sort triangles by their angle properties? by their side properties?
- Can you sort quadrilaterals into groups of rectangles, squares, and parallelograms?

You want to follow up this line of reasoning with questions about greater and less than as they pertain to negative numbers.

Share:

Group of students will share--out discovery

Summarize:

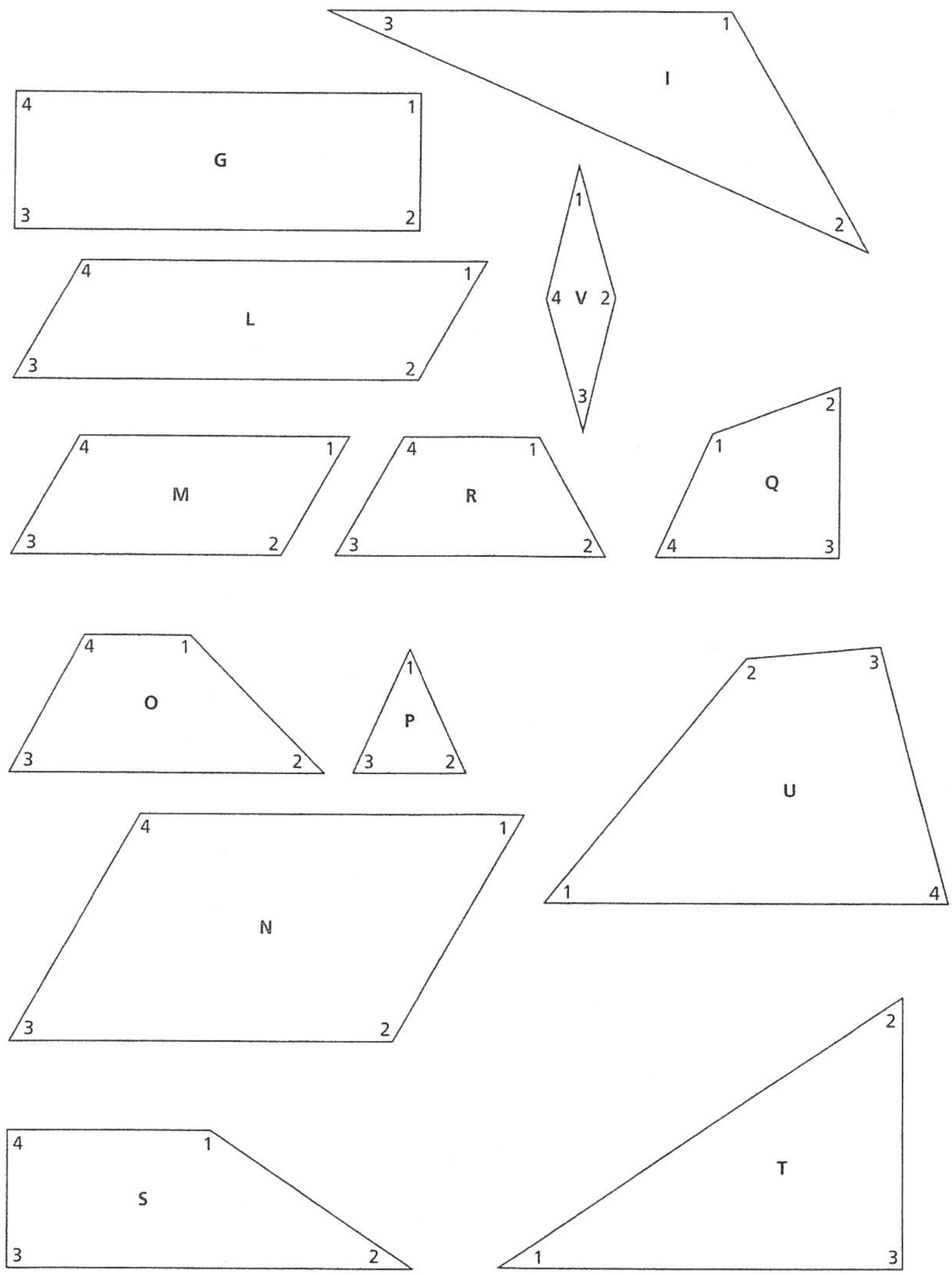
Gather interesting ways the class sorted the polygons. Ask students to describe squares and rectangles.
Questions

- How are they the same? • How are they different?
- What about the Shape L in the Shape Set? Is it a rectangle? Why or why not?
- Is a rectangle a parallelogram? Why or why not? • Is a square a rectangle? Why or why not?

Ask questions to help students observe reflection symmetry in regular polygons, isosceles triangles, kites, and rectangles.

Name .... Date .... Class ....

Shapes Set



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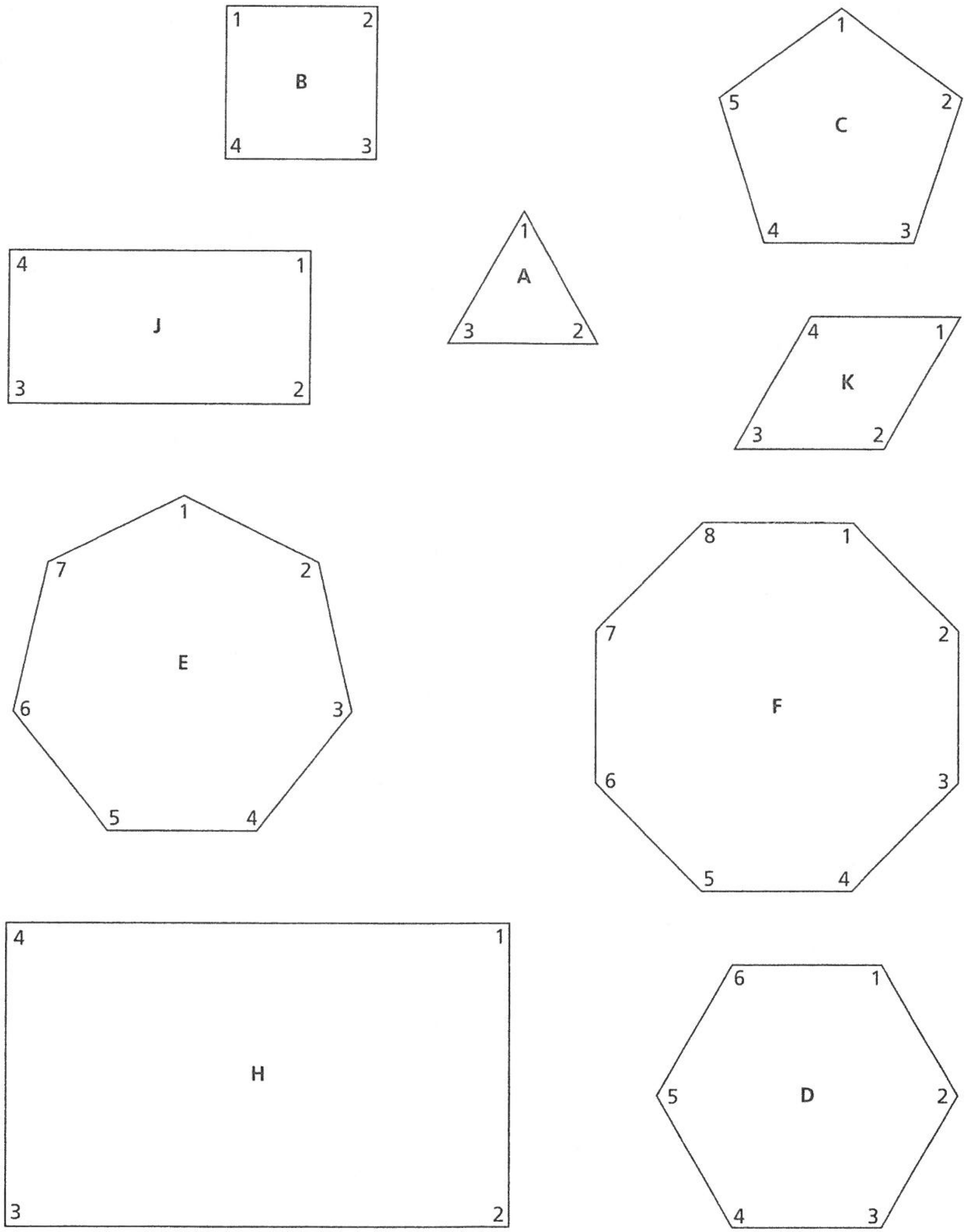
Name .... Date .... Class ....

Shapes Set



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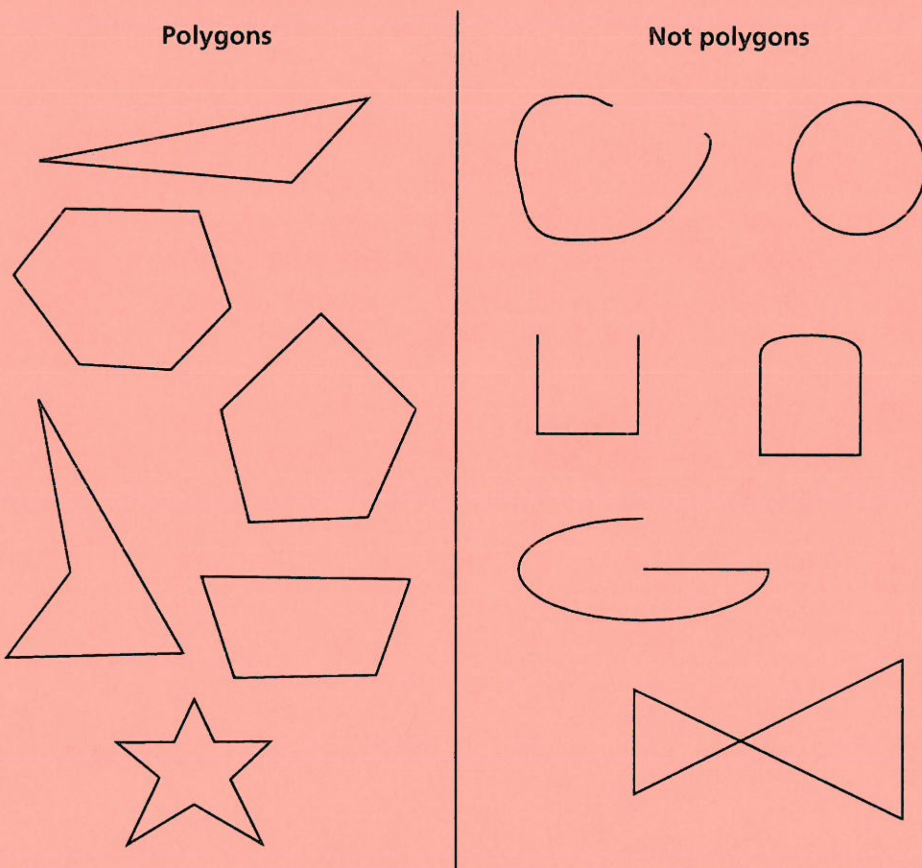
In this unit, you will investigate properties that make two-dimensional shapes useful. The unit will focus on *polygons.* First, let's review some basic concepts. A line is a familiar object. In mathematics, *line* means a straight line that has no end in either direction. You can use arrows to show that a line has no ends. A **line segment,** or *segment,* consists of two points of a line and all the points between these two points.

Getting Ready for Problem 1.1

A *polygon* is a group of line segments put together in a special way. For example, some of the shapes below are polygons and some are not.



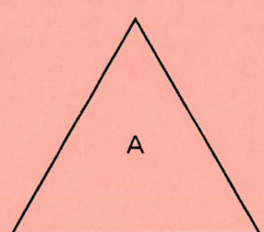
In order to be a polygon, what properties does a shape need to have?
Draw a polygon that is different from the ones above.
Then draw a shape that is not a polygon.

The line segments in polygons are called **sides.** The points where two sides of a polygon meet are called **vertices.** Polygons have special names based on the number of sides and angles they have. For example, a polygon with six sides and six angles is called a *hexagon*. The table below shows the names of some common polygons.

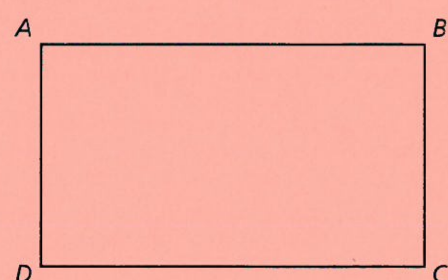
Common Polygons

Number of Sides and Angles	Polygon Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon

You can label a polygon by using a single letter or numeral for the entire shape or by marking each corner, or vertex, with a different letter. To refer to a polygon with lettered vertices, start with any letter and list the letters in order as you move around the polygon in one direction. For the rectangle below, you could say rectangle *CDAB* or rectangle *DCBA* (but not rectangle *ACDB*).



Triangle A



Rectangle *ABCD*

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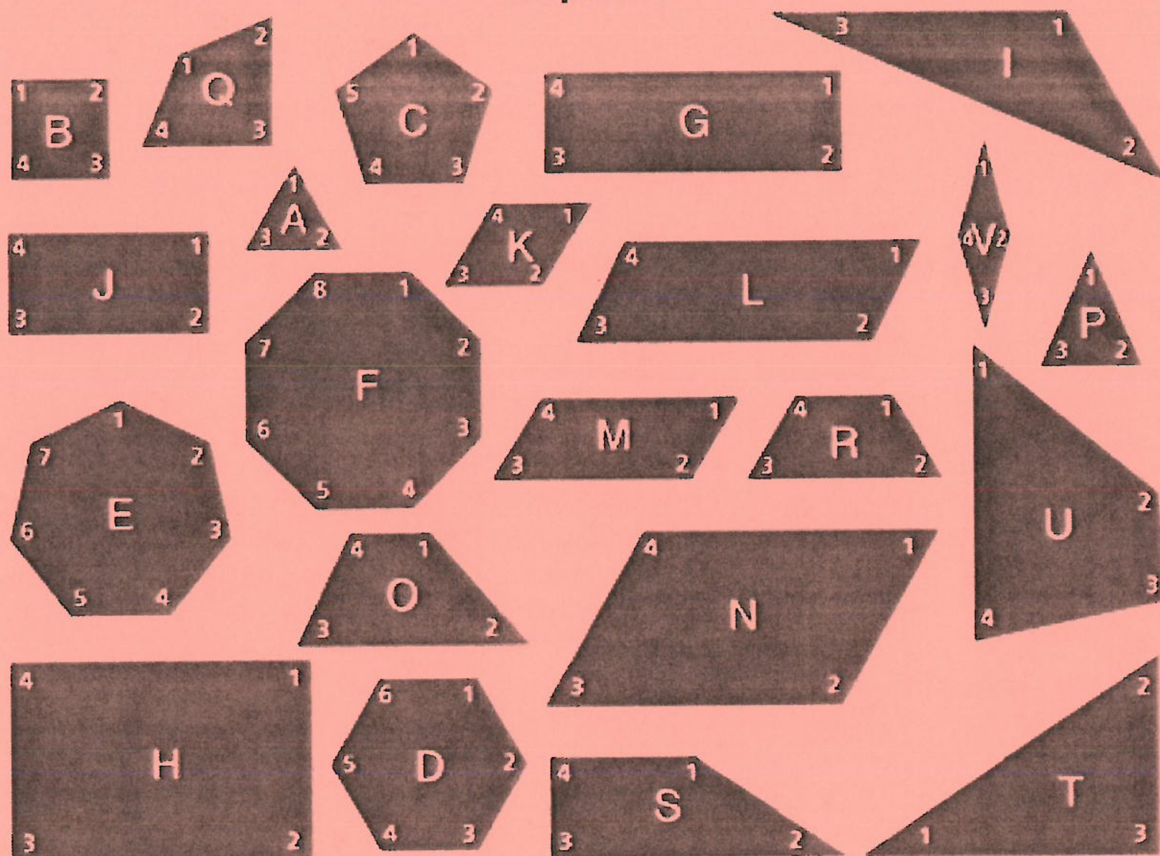
1.1 Sorting Shapes

TEKS/TAKS 6(12)A Communicate mathematical ideas using physical models. 6(13)A Make conjectures from sets of examples and nonexamples. 6(13)B Validate conclusions using mathematical properties.

Below are a variety of polygons. Many of these polygons have common properties.

Shapes Set



Problem 1.1 Sorting Shapes

A. Sort the polygons in the Shapes Set into groups so that the polygons in each group have one or more properties in common. Describe the properties the polygons have in common and give the letters of the polygons in each group.

B. Take all the triangles and sort them into two or more groups. Describe the properties you used to form the groups and give the letters of the triangles in each group.

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C. Take all the quadrilaterals and sort them into two or more groups. Describe the properties you used to form the groups and give the letters of the quadrilaterals in each group.

D. Rose put Shapes R, O, and S into the same group. What properties do these polygons have in common? Would Shape U belong to this group? Explain.

ACE **Homework starts on page 17.**

In Problem 1.1, you may have sorted the triangles according to the number of equal-length sides they have. An **equilateral triangle** has three sides the same length. An **isosceles triangle** has two sides the same length. A **scalene triangle** has no sides the same length. (The small marks on the sides of each triangle indicate sides that are the same length.)

Equilateral Triangle

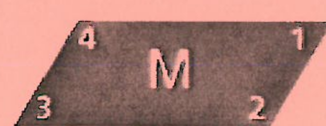
Isosceles Triangle

Scalene Triangle

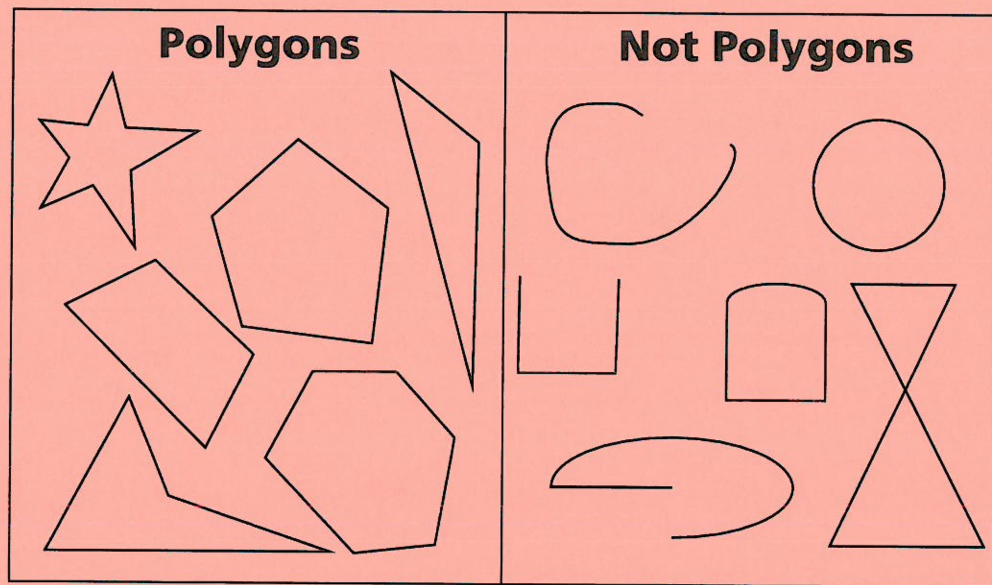
You may have sorted the quadrilaterals according to the number of sides with the same length or the number of angles of the same measure. A **square** is a quadrilateral with four sides the same length and four angles of the same measure. A **rectangle** is a quadrilateral with opposite sides the same length and four angles of the same measure. A **parallelogram** is a quadrilateral with opposite sides the same length and opposite angles of the same measure. (Note: angles 1 and 3 and angles 2 and 4 are opposite angles in the quadrilaterals below.) You will be seeing these shapes throughout this unit.

Square

Rectangle



Parallelogram



How would you describe the differences between the two groups of figures?

What test would you suggest for deciding if a figure is a polygon?

What familiar objects have shapes like the polygon examples?

What other shapes could be used as examples of polygons and non-polygons?

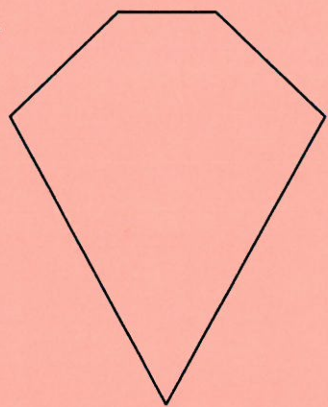
Name .... Date .... Class ....

Labsheet 1ACE

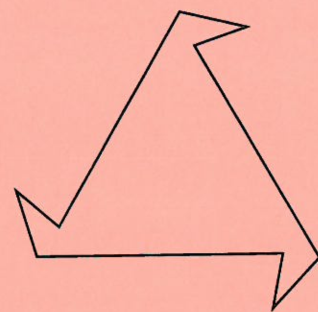
Exercise 1

1. Tell whether each figure is a polygon. Explain how you know.

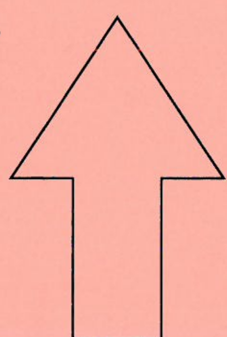
a.



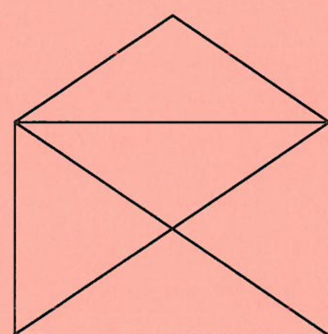
b.



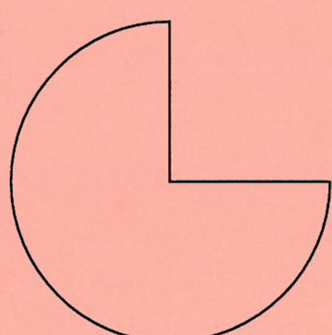
c.



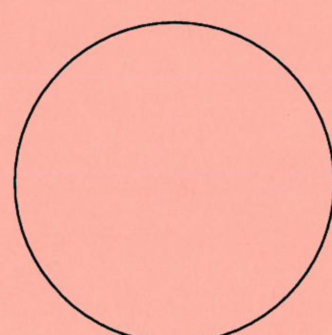
d.



e.



f.



Labsheet 1ACE

Exercise 2

Common Polygons

Number of Sides and Angles	Polygon Name	Examples in the Shapes Set
3	triangle	
4	quadrilateral	
5	pentagon	
6	hexagon	
7	heptagon	
8	octagon	
9	nonagon	
10	decagon	
12	dodecagon	

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Name .... Date .... Class ....

Labsheet 1ACE

Exercise 64

Common Quadrilaterals

Sides and Angles	Name	Examples in the Shapes Set
All sides are the same length.	rhombus	
All sides are the same length and all angles are right angles.	square	
All angles are right angles.	rectangle	
Opposite sides are parallel.	parallelogram	
Only one pair of opposite sides are parallel.	trapezoid	

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1.2 In a spin (Angles and Rotations)

Lesson 2 for 7th grade --1 day

The goal of this day's problem are to develop students understanding of angles as rotations or turns and their awareness of common benchmark angles, the multiples of 30 and 45 degrees. To help define as (1/90) th of a right angle.

Launch:

Launch this Problem by asking students what they know about the X-Games or by showing the Launch Video. We could ask Levi LaValley who graduated from Northland (Remer) to stop by or show youtube.

With 10 medals spread among six different snowmobile disciplines, LaVallee is one of the last great crossover riders. He's tied with Dave Mirra and Bob Burnquist for most unique XG disciplines from which he's medaled. At XG Aspen 2016, LaVallee could be the first three-time Snowmobile Freestyle winner, but he'll have to beat defending gold medalist Colten Moore, another two-time winner. LaVallee will also compete in SnoCross, where he medaled a decade ago, but says he is actually rooting for Kyle Pallin, the new star of his own Team LaVallee. Levi retired from the regular snocross tour in 2014 to focus on running his team but says the X Games invitations are too hard to resist.

I Show the common benchmark angles of 90°, 45°, and 180°. Refer students to the two grids in the Problem.

Questions

- **In the grid on the left, what are the measures of some of the angles? Look at angles with a vertex at the center of the grid.**
- **In the grid on the right, what are the measures of some of the angles?**
- **Using these grids, describe a point by giving two numbers. The first number tells how far to move from the center of the grid. The second number tells the amount of turn measured in degrees. How would I find the point (3, 90°)?**

Be sure students understand how the grids work before they play Four in a Row.

Explore:

As students play several games against each other, circulate to see that they are reading the coordinates correctly. Remind them to think about strategies for winning the game.

Suggested Questions

- What strategies did you discover for winning?

Look for students' ability to estimate and reason about angle size.

Share:

Group of students will share--out discovery

Summarize:

To be sure that students have the right idea about rotation and angle measurement, especially for angles larger than 90°, display copies of the two grids and ask students to show the points of coordinates.

Suggested Questions

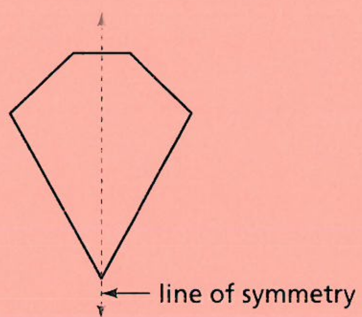
- Where are the points with coordinates (2, 135°), (3, 225°), (1, 270°), (3, 300°), etc.?

1.2 Symmetries of Shapes

TEKS / TAKS 6(12)A Communicate mathematical ideas using physical models. 6(13)A Make conjectures from sets of examples and nonexamples. 6(13)B Validate conclusions using mathematical properties.

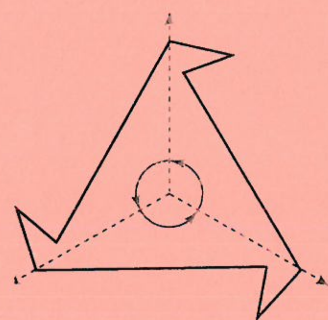
As you study the polygons in this unit, look for ways that different combinations of side lengths and angle sizes give different shapes. In particular, look for shapes that have attractive *symmetries*.

Reflection Symmetry



A shape with **reflection symmetry** has two halves that are mirror images of each other. If the shape is folded over its **line of symmetry,** the two halves of the shape match exactly.

Rotation Symmetry



If you rotate any shape a full turn, it will look like it did before you rotated it. When you rotate a shape *less* than a full turn about its center point and it looks exactly as it did before it was rotated, it has **rotation symmetry.**

In the polygon shown above, there are three places in the rotation where the polygon will look exactly the same as when you started.

Reflection symmetry is sometimes called *line* or *mirror symmetry.* (Can you see why?) Rotation symmetry is sometimes called *turn symmetry.*

Getting Ready for Problem 1.2

- Which of the following shapes have reflection symmetry?
- Which of the following shapes have rotation symmetry?

Problem 1.2 Symmetry

Use the Shapes Set from Problem 1.1.

A. Look at the triangles.

 1. Which triangles have reflection symmetry? Trace these triangles and draw all the lines of symmetry.

 2. Which triangles have rotation symmetry?

 3. Which triangles have no symmetries?

B. Look at the quadrilaterals.

 1. Which quadrilaterals have reflection symmetry? Trace these quadrilaterals and draw all the lines of symmetry.

 2. Which quadrilaterals have rotation symmetry?

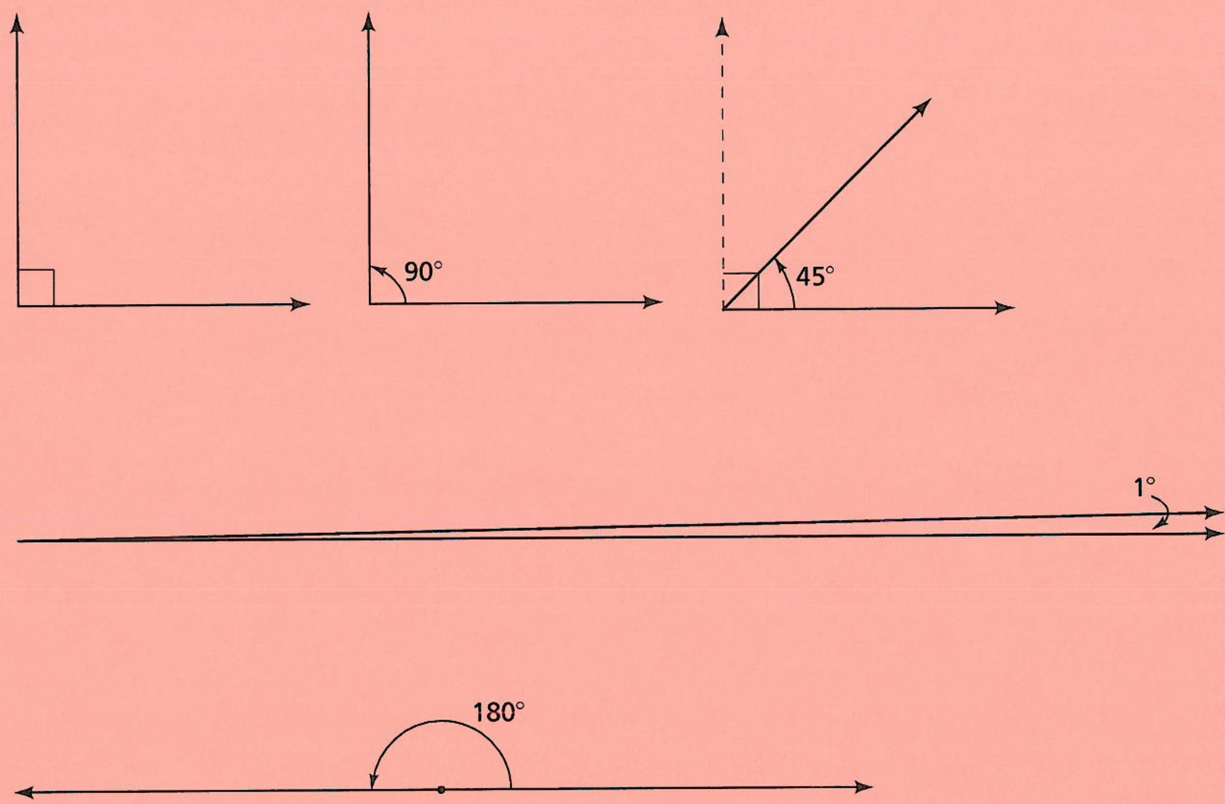
 3. Which quadrilaterals have no symmetries?

C. Look at the remaining polygons (the polygons that are not triangles or quadrilaterals). What is special about these shapes?

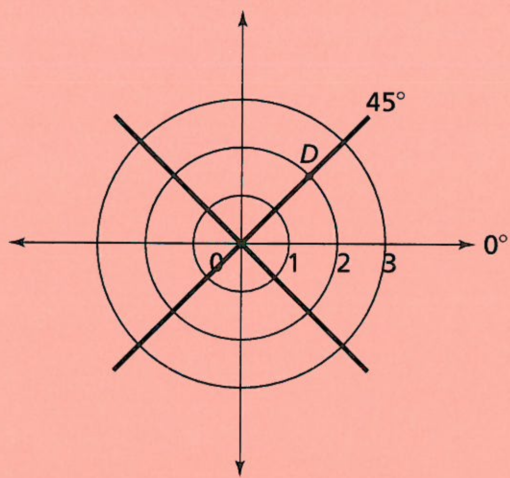
D. Find shapes with symmetry in your classroom. Sketch each shape and describe its symmetries.

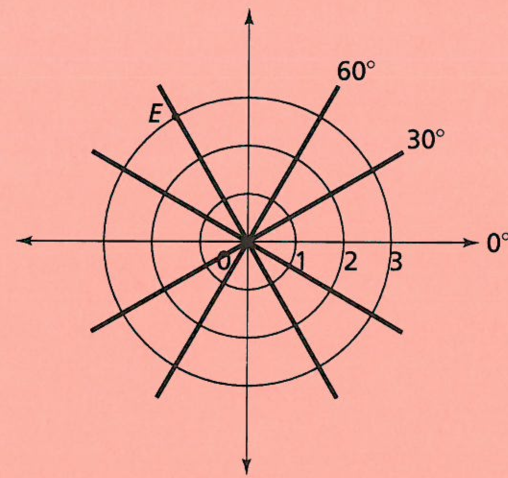
ACE Homework starts on page 17.

Problem 1.2A Angles



Problem 1.2B Circular Grids





Four in a Row Rules

Choose one of the circular grids. The grids have either 30° or 45° intervals.

- Player A chooses a point where a circle and grid line meet. Then player A says the coordinates of the point.
- Player B checks that the coordinates Player A gave are correct. If they are, Player A marks the point with an X. If they are not, Player A does not mark the point.
- Player B chooses a point and says its coordinates. If the coordinates are correct, Player B marks the point with an O.
- Players continue to take turns, saying the coordinates of points and marking the points. To win, be the first player to get four marks in a row, either along a grid line or around a circle.

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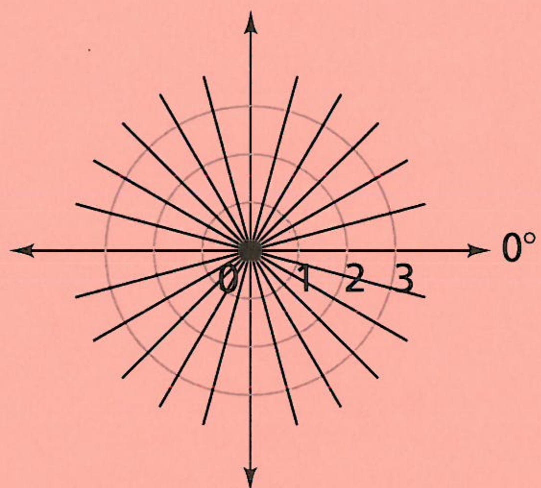
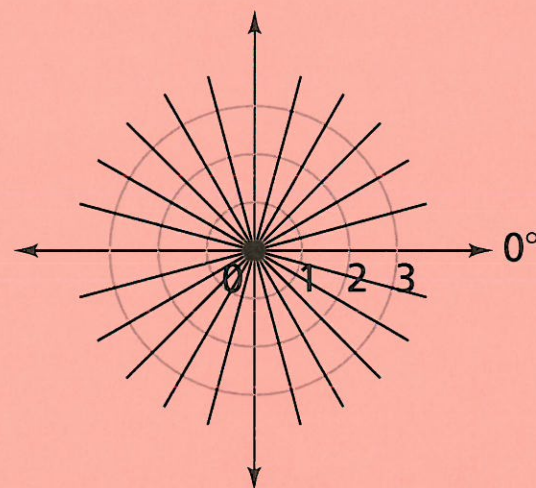
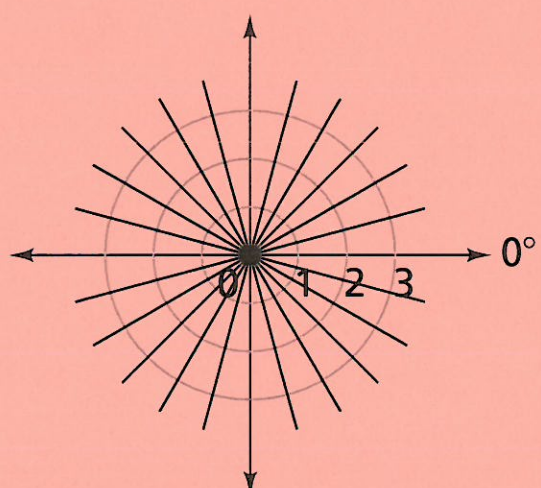
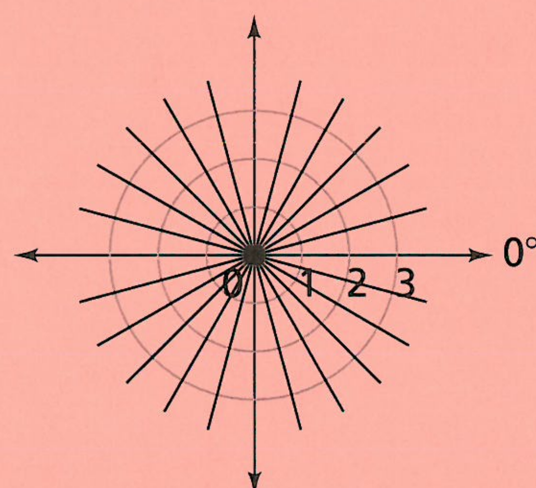
Problem 1.2D
15° Grids









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1.3 Estimating Measure of Rotations and Angles

Lesson 3 for 7th grade --1 day

To describe and measure rotations, think of angles as a rotation or turn and not simply as two rays with a common endpoint, there are two rotation angles. The objective of this problem is confront directionality and to build estimation skills for angles measurement. We will also review or introduction, depending on a student's prior experiences, of the conventions for labeling angles.

Launch

Begin with the Focus Question by discussing the issues of estimating the measure of rotation on an unlabeled drawing of an angle. Students might assume that you mean the counterclockwise rotation angle starting from the ray pointing to the right.

Questions

- **How do you know that the rotation angle I had in mind wasn't the one that started from the other ray?**
- **What would be the measure of that rotation angle?**

It is important to be clear which ray is the initial side and which ray is the terminal side. On a drawing, using an arc with an arrowhead at the end of the rotation is one way to distinguish between the initial and terminal sides.

Explore

After making the point that direction matters in rotation, emphasize that the convention in mathematics is to use a counterclockwise direction if one is not specified. Also, make the labeling convention for angles clear. Look for strategies that students are using to sketch the angles. Use these questions to help students with their angle sketches.

Questions

- **Is the angle greater than, less than, or equal to 90?**
- **Is it greater than 90? Is it less than 180?**
- **Is it greater than 180? Is it less than 270? Etc.**

Share:

Have small group share finding.

Summarize

Use the Summarize to determine whether students can estimate the measure of a directed angle given a picture and whether they can sketch and label an angle when given its measure or a description of the amount of turn involved. This is a good time to review acute (less than 90), obtuse (greater than 90 but less than 180) and right (90) angles with your students.

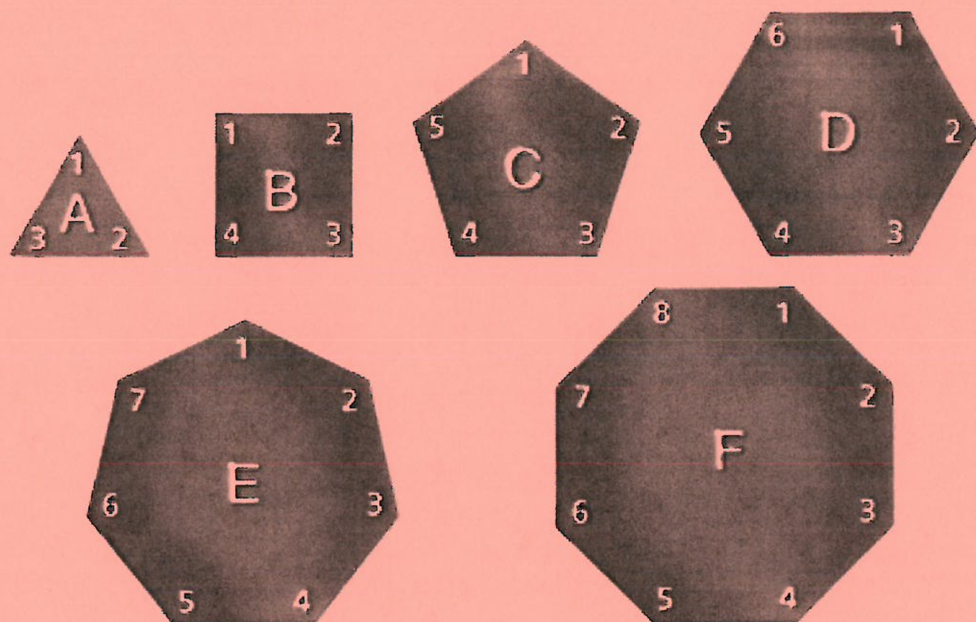
Questions

- **In Questions C and D, which angles are acute, obtuse, or right?**
- **Which angles in the room are acute, obtuse or right?**

1.3 Tiling a Beehive

TEKS / TAKS **6(12)A** Communicate mathematical ideas using physical models.
6(13)A Make conjectures from patterns.

A **regular polygon** is a polygon in which all the sides are the same length and all the angles have the same measure. In an **irregular polygon,** all sides are *not* the same length or all the angles are *not* the same measure. The shapes below are regular polygons.



You can find an interesting pattern of regular hexagons on the face of a honeycomb. The hexagons fit together like tiles on a floor.

Tiling means covering a flat surface with shapes that fit together without any gaps or overlaps.

Which regular polygons can be used to tile a surface?

Problem 1.3 Tiling With Regular Polygons

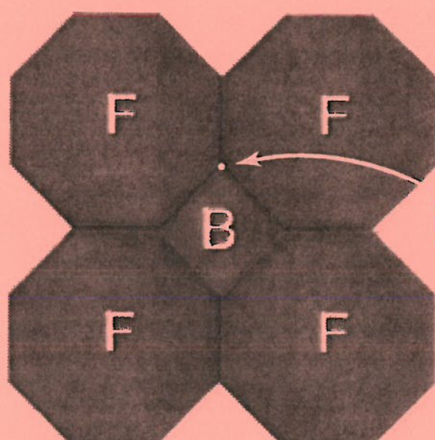
Use Shapes A–F from your Shapes Set or cutouts of those shapes. As you work, try to figure out why some shapes cover a flat surface, while others do not.

A. **1.** First, form tile patterns with several copies of the *same* polygon. Try each of the regular polygons. Sketch your tilings.

2. Which regular polygons fit together, without gaps or overlaps, to cover a flat surface?

B. Next, form tile patterns using combinations of two or more different shapes. Sketch your tilings.

C. The following tiling may be one that you found. Look at a point where the vertices of the polygons meet.



At this vertex, two octagons and one square fit together.

active math online
For: Tessellation Activity
Visit: PHSchool.com
Web Code: amd-3103

1. Look back at each tiling you made. Find a point on the tiling where the vertices of the polygons meet.

2. Describe exactly which polygons fit around this point and the pattern of how they fit together.

3. Is this pattern the same for all other points where the vertices of the polygons meet in this tiling?

ACE Homework starts on page 17.

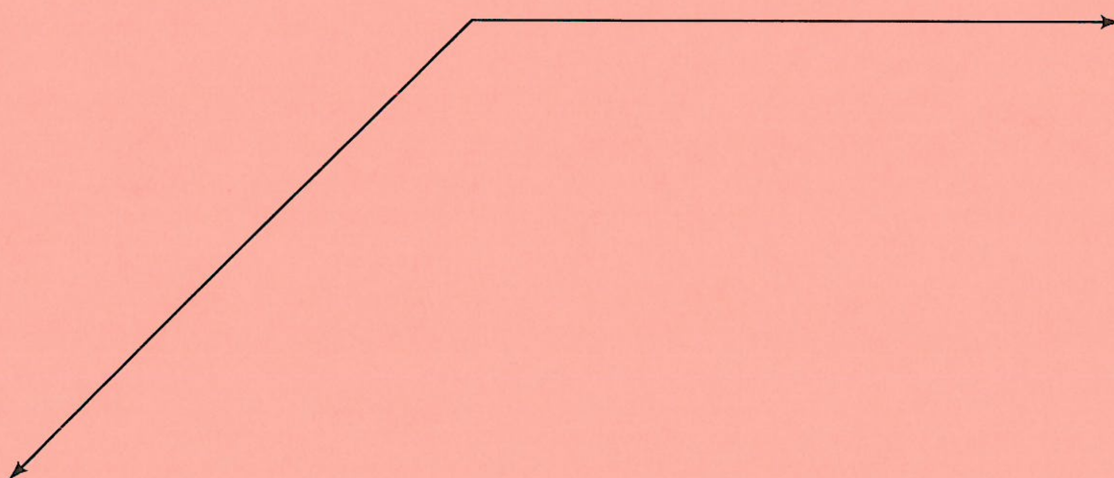
Did You Know?

Tilings are also called *tessellations*. Artists, designers, and mathematicians have been interested in tessellations for centuries. The Greek mathematician and inventor Archimedes (c. 287–212 B.C.) studied the properties of regular polygons that tiled the plane. Beginning in the middle of the eighth century, Moorish artists used tessellating patterns extensively in their work.

The Dutch artist M.C. Escher (1898–1972), inspired by Moorish designs, spent his life creating tessellations. He altered geometric tessellating shapes to make birds, reptiles, fish, and people.

Go Online
PHSchool.com
For: Information about tessellations
Web Code: ame-9031

Problem 1.3A Measure Rotation



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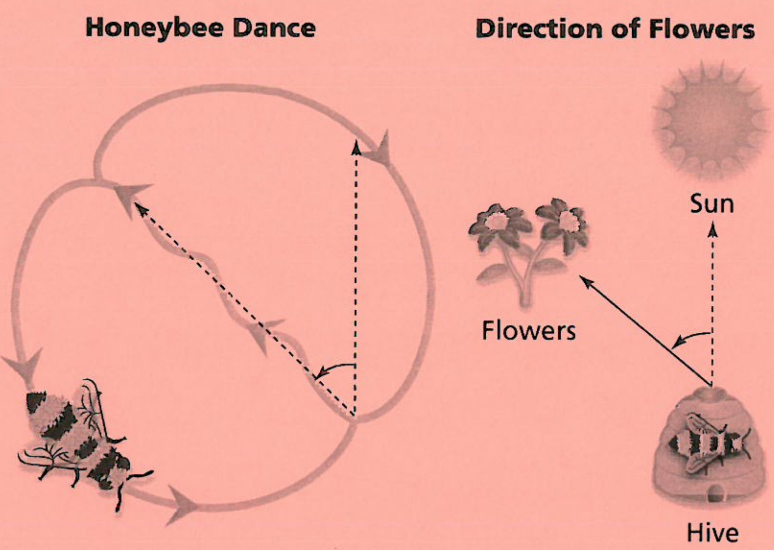
Problem 1.3B Honeybee Dance



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1.4 Measuring Angles

Lesson 4--7th grade 2 days to complete

The goal of this Problem is to develop student understanding of two standard tools for angles measurement - the goniometer or angle ruler and the protractor. Some student like the goniometer better than common protractor.

Launch

Draw an angle on the board or overhead and ask students to estimate its measure. First, have students look at the indicated shapes as you ask questions:

Questions

- Which of these shapes has the smallest angle measure?
- Which shape has the angle with the largest measure?
- Can you find a 90 angle in one of these shapes? How can you tell it is 90?
- How do the angles of Shape D compare to a right angle? Are their measures greater than or less than a right angle?

Once students are ready, demonstrate how to use the angle ruler.

Explore

As students use an angle ruler in Questions A–C, monitor their work to be sure that they are placing the angle ruler correctly and reading the measures accurately.

Suggested Questions

- Where should the rivet be placed on the angle?
- Which arm remains stationary over the initial side?
- Which arm do you rotate counterclockwise to the terminal side?

Students should record how they estimate the size of the angles in the shapes. Remind them to sketch each shape and to label the angle measures on the shape.

Share

Have students report their findings in a class discussion. Take time to explore all the strategies they used to arrive at their answers.

Summarize

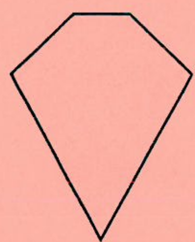
Questions

- How do you decide the number of degrees for angles between the 5 intervals on the angle ruler?
- What things do you check to make sure you are making accurate measurements?
- Did everyone get the same answer for Question E? Why?

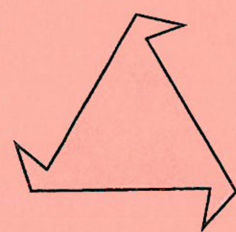
Applications

1. Tell whether each shape is a polygon. If it is, tell how many sides it has.

a.

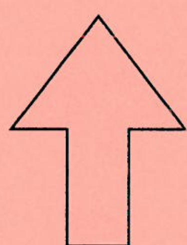


b.



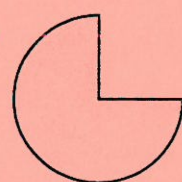
2. Tell whether each figure is a polygon. Explain how you know.

a.

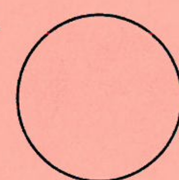


b.

c.



d.



3. Examine Shapes L, R, and N from the Shapes Set (or look at the drawings from Problem 1.1). How are these polygons alike? How are they different?

4. Examine Shapes A, I, P, and T from the Shapes Set (or look at the drawings from Problem 1.1). How are these polygons alike? How are they different?

5. Tell whether each statement is *true* or *false*. Justify your answers.

a. All squares are rectangles.

b. All rectangles are parallelograms.

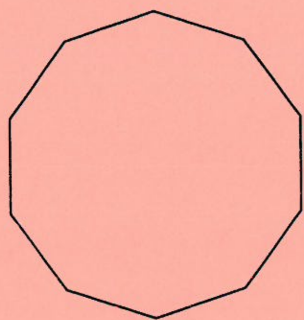
c. All rectangles are squares.

d. There is no rectangle that is not a parallelogram.

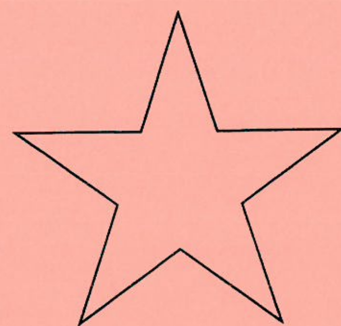
Homework Help Online
PHSchool.com
For: Help with Exercise 6
Web Code: ame-3106

6. a. Copy the shapes below. Draw all the lines of symmetry.

Shape 1

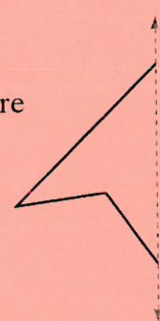


Shape 2



b. Do these shapes have rotation symmetry? Explain.

7. Half of the figure below is hidden. The vertical line is a line of symmetry for the complete figure. Copy the part of the figure shown. Then draw the missing half.

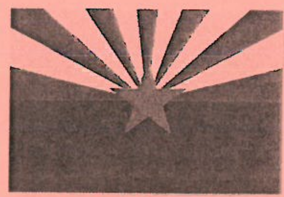


8. Below is a rug design from the Southwest United States.

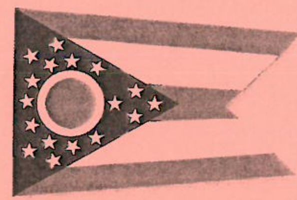
a. Name some of the polygons in the rug.

b. Describe the symmetries of the design.

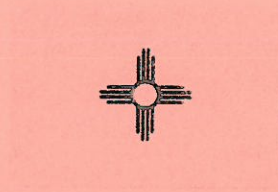
9. Here are three state flags.



Arizona



Ohio



New Mexico

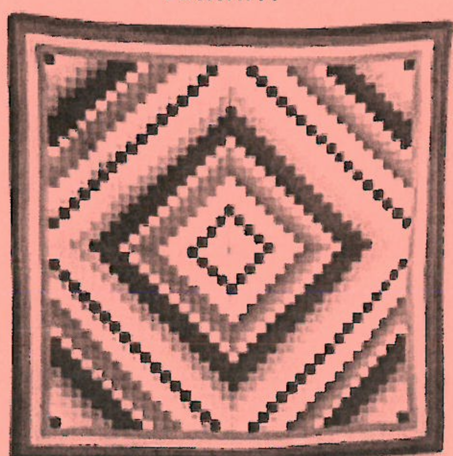
a. Describe the lines of symmetry in each whole flag.

b. Do any of the shapes or designs within the flags have rotation symmetry? If so, which ones?

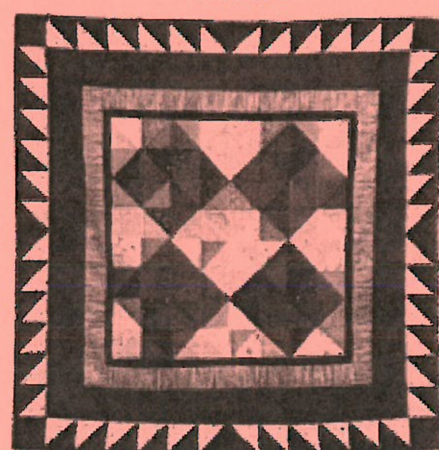
c. Design your own flag. Your flag should have at least one line of symmetry. Your flag should also include three shapes that have rotation symmetry. List the shapes in your flag that have rotation symmetry.

For Exercises 10 and 11, use these quilt patterns.

Pattern A



Pattern B



10. Name some of the polygons in each quilt pattern.

11. Describe the symmetries of each quilt pattern.

12. a. Does a circle have any symmetries? If so, explain and show some examples.

b. Can you make a tiling pattern with circles? If so, explain and show some examples.

13. Choose a rectangle from your Shapes Set, or draw your own. Find two ways that copies of your rectangle can be used to tile a surface. Sketch your tilings.

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14. Choose a parallelogram from your Shapes Set, or draw your own. Find two ways that copies of your parallelogram can be used to tile a surface. Sketch your tilings.

15. Choose a scalene triangle from your Shapes Set, or draw your own. Find two ways that copies of your triangle can be used to tile a surface. Sketch your tilings.

16. Find three examples of tilings in your school, home, or community. Describe the patterns and make a sketch of each.

Connections

For each fraction, find two equivalent fractions. One fraction should have a denominator less than the one given. The other fraction should have a denominator greater than the one given.

17. $\frac{4}{12}$ **18.** $\frac{9}{15}$ **19.** $\frac{15}{35}$ **20.** $\frac{20}{12}$

Copy the fractions and insert <, >, or = to make a true statement.

21. $\frac{5}{12}$ $\frac{9}{12}$ **22.** $\frac{15}{35}$ $\frac{12}{20}$ **23.** $\frac{7}{13}$ $\frac{20}{41}$ **24.** $\frac{45}{36}$ $\frac{35}{28}$

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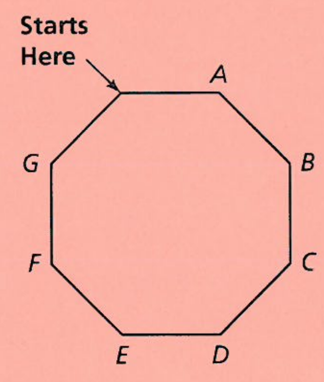
25. Multiple Choice Choose the correct statement.

A. $\frac{5}{6} = \frac{11}{360}$ **B.** $\frac{3}{4} = \frac{300}{360}$ **C.** $\frac{1}{4} = \frac{90}{360}$ **D.** $\frac{3}{36} = \frac{33}{360}$

Alberto's little sister Marissa decides to take a ride on a merry-go-round. It is shaped like the one shown. Marissa's starting point is also shown.



26. Multiple Choice Choose the point where Marissa will be after the ride completes $\frac{4}{8}$ of a full turn.

F. point C **G.** point D

H. point E **J.** point G

27. Multiple Choice Where will Marissa be after the ride completes $\frac{1}{2}$ of a full turn?

A. point B **B.** point C

C. point D **D.** point F

28. Find all the possible rectangles with whole number side lengths that can be made with the given number of identical square tiles. Give the dimensions of each rectangle.

a. 30 square tiles

b. 24 square tiles

c. 36 square tiles

d. 17 square tiles

e. How are the dimensions of the rectangles for a given number of square tiles related to factor pairs of the number?

f. Which of the rectangles you made were squares? Give the dimensions of the squares you made.

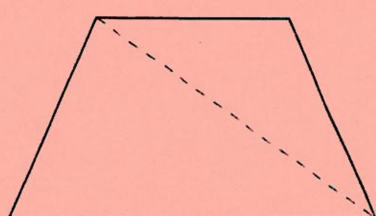
Extensions

For Exercises 29–33, one diagonal of each quadrilateral has been drawn. Complete parts (a) and (b) for each quadrilateral.

a. Is the given diagonal a line of symmetry? Why or why not?

b. Does the figure have any other lines of symmetry? If so, copy the figure and sketch the symmetry lines.

29.



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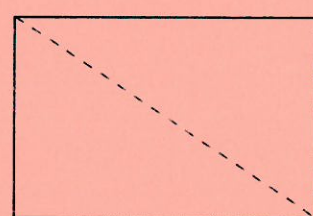

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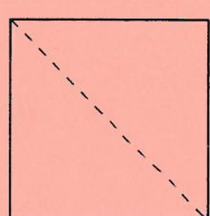

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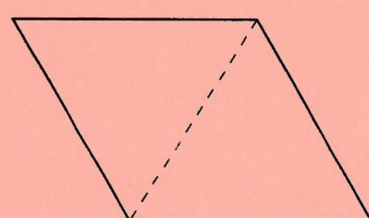

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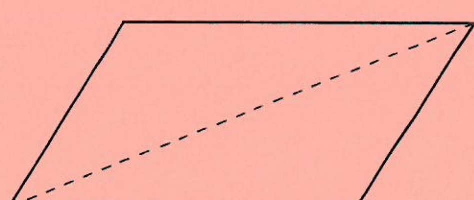
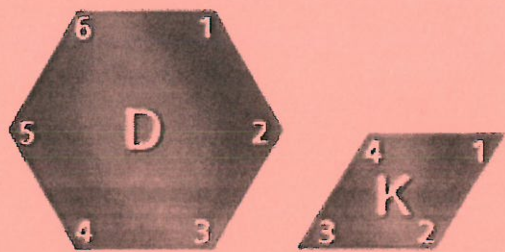


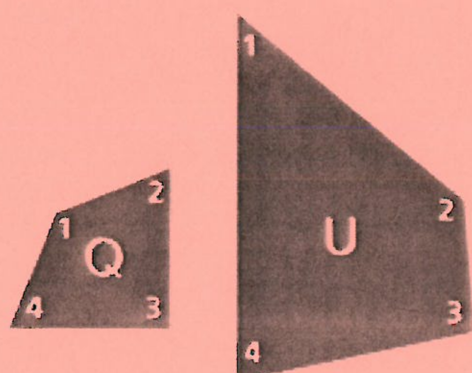
For Exercises 34 and 35, use the two given shapes to form a tiling pattern. Trace and cut out the shapes, or use shapes from your Shapes Set. Sketch your tilings.

34.



35.

36. Choose an irregular quadrilateral from your Shapes Set, such as Q or U, or draw your own. Cut out several copies of your quadrilateral. See whether you can use the copies to tile a surface. Sketch your findings. Test other irregular quadrilaterals to see if they can be used to tile a surface. Summarize what you find about using irregular quadrilaterals to tile a surface.



Mathematical Reflections 1

In this investigation, you explored some properties of polygons. You saw that some polygons have reflection or rotation symmetry, while others have no symmetries. You also discovered that some polygons fit together like tiles to cover a flat surface, while others do not. These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. a. What does it mean for a figure to have reflection symmetry? Give an example.

b. What does it mean for a figure to have rotation symmetry? Give an example.

2. a. Using one shape at a time, which regular polygons will fit together to tile a surface? Which regular polygons cannot tile a surface?

b. Why do you think that some shapes make tilings and some do not?

Unit Project What's Next?

What information about shapes can you add to your *Shapes and Designs* project?

Finding Pythagorean Theorem Day 1

During this lesson, eighth grade students will be introduced to the Pythagorean Theorem and in Geometry students will review : $a^2 + b^2 = c^2$. They will construct a right triangle on graph paper and draw squares on each side of the triangle.

Minnesota Standards

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Goal:

At the end of this lesson, students should know and be able to explain the attributes of a right triangle. They should be able to apply their new found knowledge of the Pythagorean Theorem to a real life scenario.

Students will be able to: • Find the length of a segment without using the Pythagorean theorem • Be able to describe the Pythagorean theorem in terms of area • Prove the Pythagorean theorem • Use the Pythagorean theorem to find the length of a segment • Be able to prove if a triangle is an acute, right or obtuse triangle • Find uses outside of the classroom for the Pythagorean theorem.

Day One – Calculating the Length of a Segment Using Area

Objective:

Students will learn how to calculate the length of a segment using the area of squares and triangles. Students will learn how to use a Geoboard to calculate the lengths of a segment without using the Pythagorean Theorem. Students will also start to visualize the concept of using area of squares and triangles to prove the Pythagorean Theorem.

Launch:

To begin class, have the students find the area of several squares and triangles that are on the blackboard. If it is necessary review the formula for the area of a square and triangle.

Geoboards and geopaper will be handed out. Using the overhead geoboard I will demonstrate to the students the process that I want them to follow.

1) Use a rubber band to create a right angle on the geoboard.

2.) How will you find all the different lengths, How many lengths are there? When will we have them all. How do we find out how long each length is?

3.)Use a rubber band to create square on all sides of the right triangle geoboard.

4.) Can we find the area of each square? How?

Explore:

Geoboards and geopaper will be handed out. Using the overhead geoboard I will demonstrate to the students the process that I want them to follow.

1) Use a rubber band to create a right angle on the geoboard.

2.) How will you find all the different lengths, How many lengths are there? When will we have them all. How do we find out how long each length is?

3.)Use a rubber band to create square on all sides of the right triangle geoboard.

4.) Can we find the area of each square? How?

Share

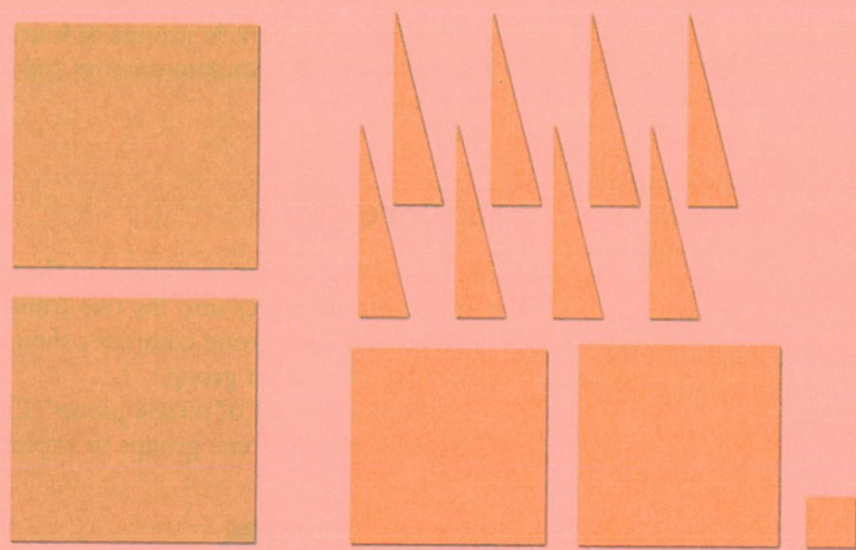
Have groups share what area and lengths they found and how the go together.

Summerize:

When groups have finished the problem, ask about any general patterns they noticed. Some may mention the relationship between the squares and the sides of the right triangle. Others may notice that a side length of a puzzle frame is equal to the sum of the lengths of the two legs of each triangle. Demonstrate these relationships at the overhead.

Problem 3.2 A Proof of the Pythagorean Theorem

Use the puzzles your teacher gives you.



A. Study a triangle piece and the three square pieces. How do the side lengths of the squares compare to the side lengths of the triangle?

B. **1.** Arrange the 11 puzzle pieces to fit exactly into the two puzzle frames. Use four triangles in each frame.

2. What conclusion can you draw about the relationship among the areas of the three squares?

3. What does the conclusion you reached in part (2) mean in terms of the side lengths of the triangles?

4. Compare your results with those of another group. Did that group come to the same conclusion your group did? Is this conclusion true for all right triangles? Explain.

C. Suppose a right triangle has legs of length 3 centimeters and 5 centimeters.

1. Use your conclusion from Question B to find the area of a square drawn on the hypotenuse of the triangle.

2. What is the length of the hypotenuse?

D. In this Problem and Problem 3.1, you explored the Pythagorean Theorem, a relationship among the side lengths of a right triangle. State this theorem as a rule for any right triangle with leg lengths a and b and hypotenuse length c.

ACE **Homework starts on page 38.**

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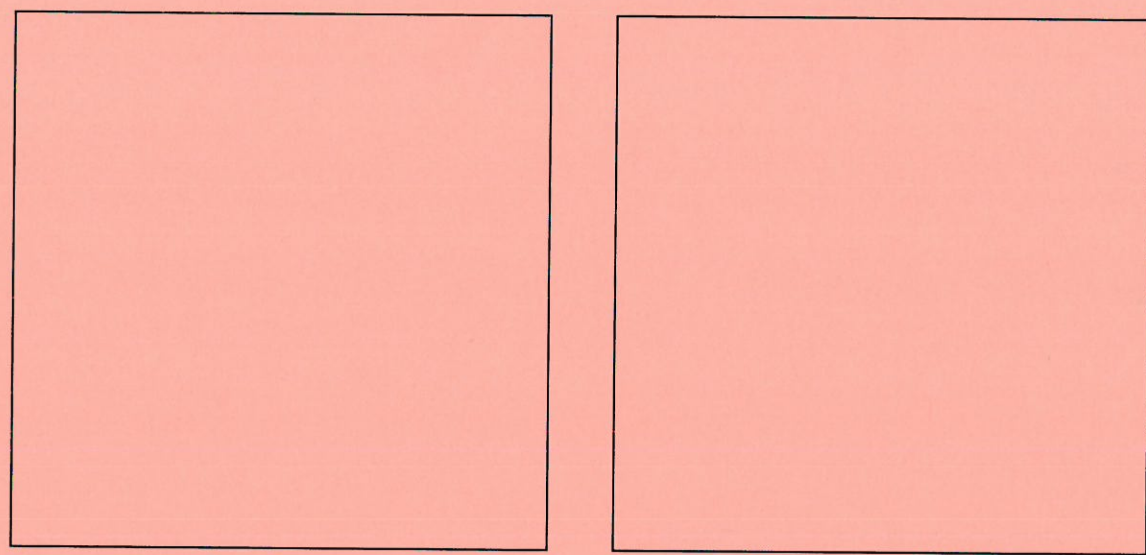
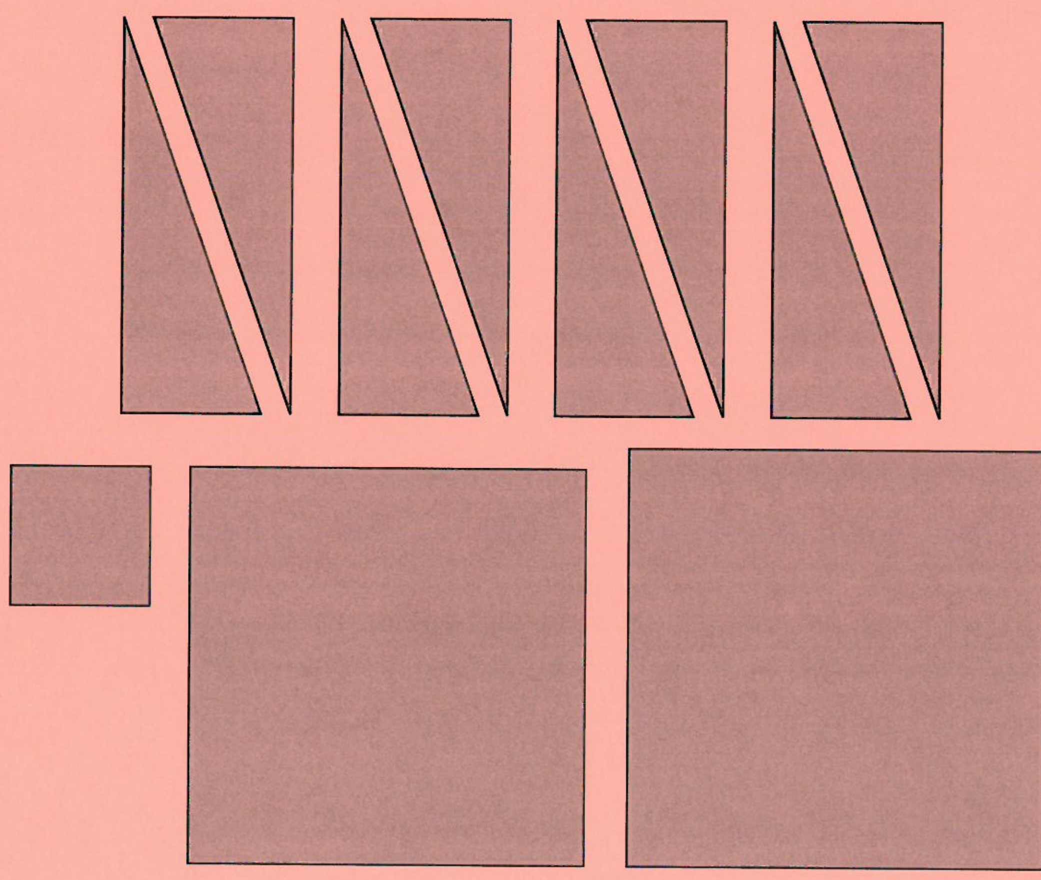
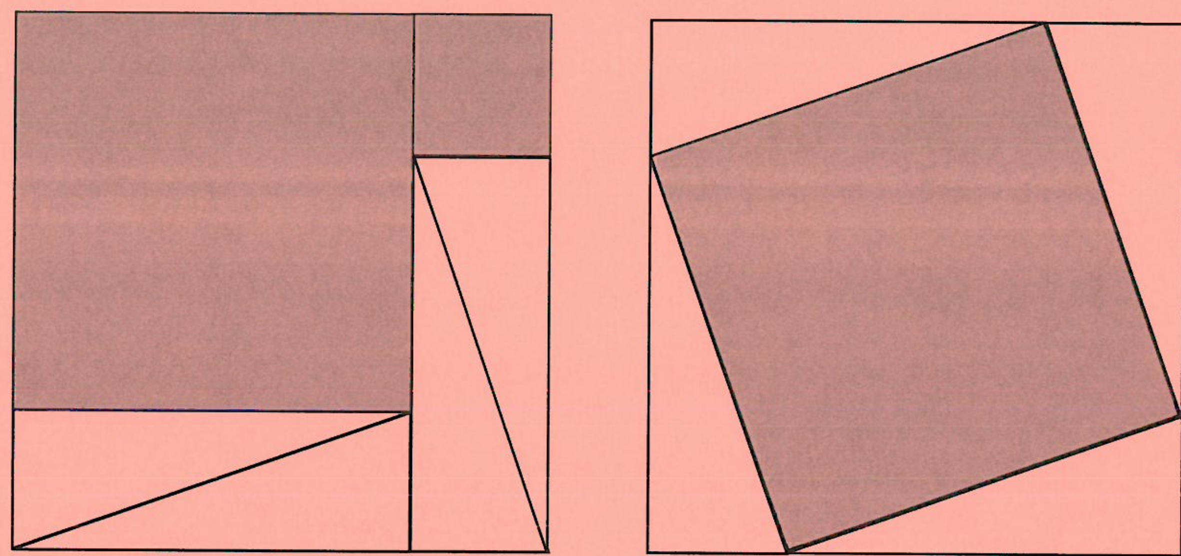
Puzzle Frames



Puzzle Pieces



Solution



Lesson: Angles in Polygons

Objective: Students will be able to draw and classify polygons based on the number of sides. Students will be able to measure the interior and exterior angles and develop an understanding of properties of the interior and angles of polygons.

Standards:

Launch: We will start with a discussion about the different classifications of polygons (including by sides, regular or irregular, concave or convex). They will be familiar with triangles and perhaps quadrilaterals, but I will challenge them to find the sum of the interior and exterior angles angles in a 20-gon and a method for finding the same angles in any polygon (n-gon), regardless of the number of sides.

Explore: Students will be working in pairs with a ruler and protractor and draw each of the different polygons, one polygon per side of paper (sides from 4-8, all must be convex and irregular). The will measure all interior and exterior angles and record the results on the drawing. Then they will make a chart with the sum of the interior angles and the sum of the exterior angles for the polygons.

Share: Students will share their results by posting their drawings around the room for all to see. They will be asked to identify a pattern in the results and try to use that pattern to find the sum of the interior and exterior angles in a 20-gon and in any polygon (n-gon). Each of the groups will be asked to share their results with the class.

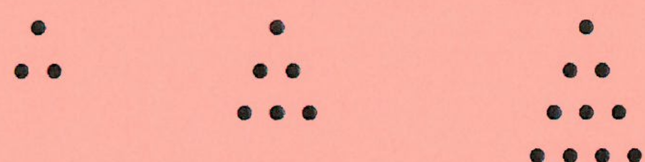
Summarize: The instructor will facilitate a class discussion in order to bring everyone to a consensus, that being that all polygons have exterior angles whose sum is 360 degrees and any n-gon has the sum of the interior angles being (n-2)180 degrees.

Lesson on Triangle and Square numbers

Goals:
Theoretic Questions
The main goal of number theory is to discover interesting and unexpected relationships between different sorts of numbers and to prove that these relationships are true. In this section we will describe a few typical number theoretic problems, some of which we will eventually solve, some of which have known solutions too difficult for us to include, and some of which remain unsolved to this day.
Sums

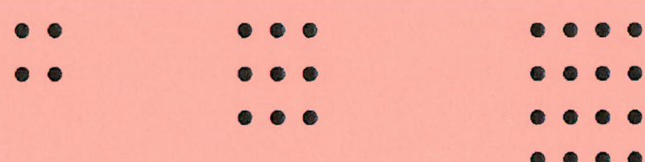
Launch:
Show students the first 3 or 4 triangle and then Square number on graph paper.
Questions
What are the patterns, Can you find a way to get the next number, What about the N? Number?
triangular 1; 3; 6; 10; 15; 21……square 1; 4; 9; 16; 25; 36; : : :
Can you draw the first 6 of each type? Numbers? Are there pentagonal, Hexagonal, ect.. Can you drawn or find them?
<u>Explore:</u>
Give out lab sheet with samples and place to graph with other Questions

Number Shapes. The square numbers are the numbers 1, 4, 9, 16, … that can be arranged in the shape of a square. The triangular numbers are the numbers 1, 3, 6, 10, … that can be arranged in the shape of a triangle. The first few triangular and square numbers are illustrated in Figure 1.1.



$1+2=3$ $\quad$ $1+2+3=6$ $\quad$ $1+2+3+4=10$

Triangular numbers



$2^2=4$ $\quad$ $3^2=9$ $\quad$ $4^2=16$

Square numbers

Now you need to graph these numbers. What do you see

Share

Summerize:

1.) Display a spreadsheet program and check for background knowledge of students,
2.) Have students enter the new data into the spreadsheets and create a graph of the information and discuss the new graphs. Are the graph discrete or continues ? Why?

Squares I. Can the sum of two squares be a square? The answer is clearly “YES”; for example 32 + 42 = 52 and 52 + 122 = 132. These are examples of Pythagorean triples. We will describe all Pythagorean triples in another lesson

Numbers That Form Interesting Shapes
A natural question to ask is whether there are any triangular numbers that are also square numbers (other than 1). The answer is “YES,” the smallest example being 36 = 62 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8: So we might ask whether there are more examples

Day 1 Building Triangles

Minnesota Standard:

_____ 9.3.3.3 Know and apply properties of equilateral, isosceles and scalene triangles solve problems and logically justify results.*For example*: Use the triangle inequality to prove that the perimeter of a quadrilateral is larger than the sum of the lengths of its diagonals.

What combinations of three side lengths can be used to make a triangle? How many different shapes are possible for such a combination of side lengths?

Launch:

Initiate work on the Investigation by asking students where they have seen triangles used in structures.When using the polystrips to represent the length, they need to be aware that it is the space between two holes that represents a length of one unit.

Questions:

• What is it that makes triangles so common in construction? • Are there special conditions for the side lengths of a triangle? • Can a triangle be built with any three side lengths?

Explore/Share:

As you monitor student work with polystrips, keep in mind a key goal of the experimentation is to help them discover the triangle inequality (the sum of any two side lengths must always be greater than the third side length).

Questions:

• Is it possible to make more than one triangle with any three side lengths? Is there a limit of the number triangle we can make?

• Can you make two triangles with corresponding sides the same length?

Summarize/Share:

The most important result from this Problem is to answer the question, "What combinations of side lengths can and cannot be used to make a triangle?"

Questions:

• Can we come up with a summary statement that would help someone, who is not here today, know how to judge whether three lengths will make a triangle without actually building the triangle? • Which sets form a triangle? Why?

• Do any of these triangles have special properties? Describe them.

Students should leave this problem with the ability to respond to the questions posed in the Problem.

Can we come up with properties of equilateral, isosceles and scalene triangles .

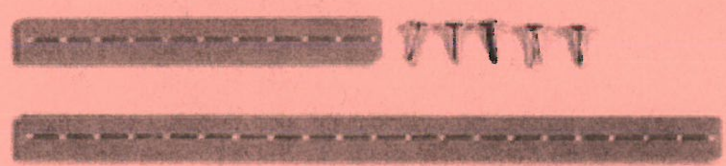
Investigation 4

Building Polygons

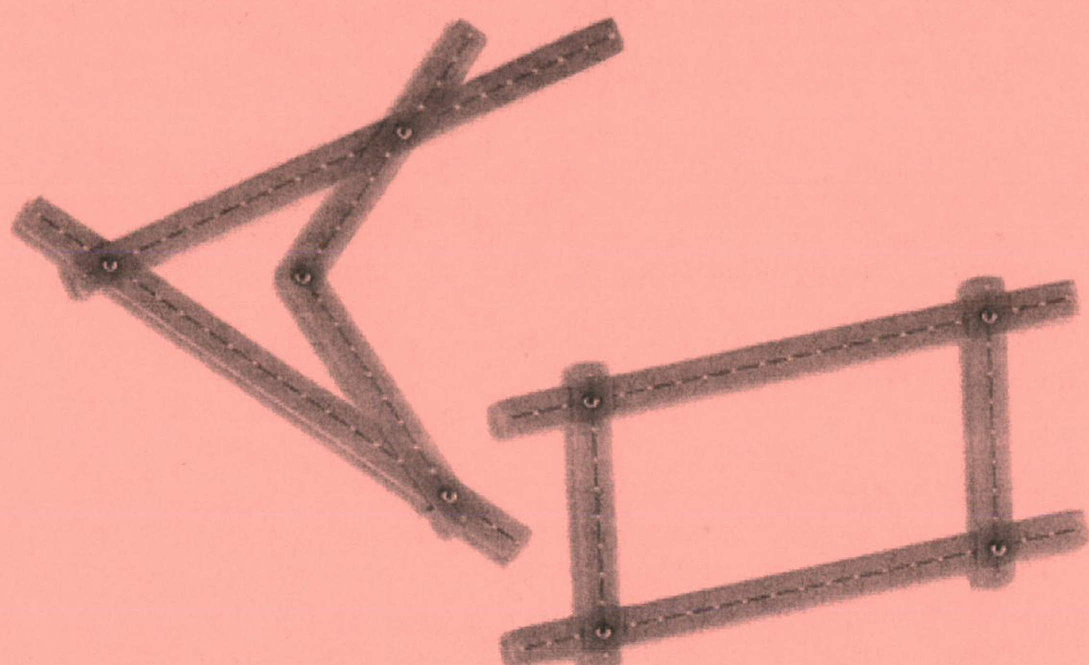
In the last two investigations, you explored the relationship between the number of sides of a polygon and the measure of its interior angles. Now you will turn your attention to the sides of a polygon.

How do the side lengths of a polygon affect its shape?

You can use polystrips and fasteners like these:



to build polygons with given side lengths and study their properties.



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4.1 Building Triangles

TEKS / TAKS 6(11)C Select an appropriate problem-solving strategy including looking for a pattern.
6(13)A Make conjectures from patterns.

Bridges, towers, and other structures contain many triangles in their design. *Why are triangles used so frequently in construction?*

Problem 4.1 Building Triangles

Make a triangle using the steps below. Sketch and label your results.

Step 1 Roll three number cubes and record the sum. Do this two more times, so that you have three sums.

Step 2 Using polystrips, try to make a triangle with the three sums as side lengths. If you can build one triangle, try to build a different triangle with the same side lengths.

Repeat Steps 1 and 2 to make several triangles.

A. **1.** List each set of side lengths that did make a triangle.

2. List each set of side lengths that did not make a triangle.

3. What pattern do you see in each set that explains why some sets of numbers make a triangle and some do not?

4. Use your pattern to find two new sets of side lengths that will make a triangle. Then find two new sets of side lengths that will not make a triangle.

B. Can you make two different triangles from the same three side lengths?

C. Why do you think triangles are so useful in construction?

ACE **Homework starts on page 76.**

Name .... Date .... Class ....

Labsheet 3.1

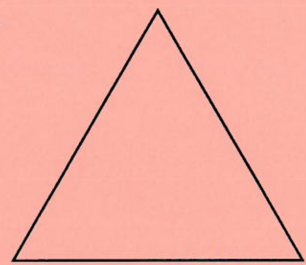
Building Triangles

Side 1	Side 2	Side 3	Triangle? (yes or no)	Sketch	Different Shape? (yes or no)

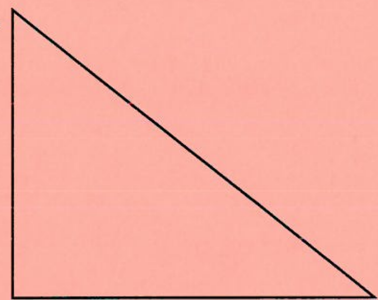
Labsheet 3.2 Question B

B. Write the shortest possible message to tell how to draw each triangle below.

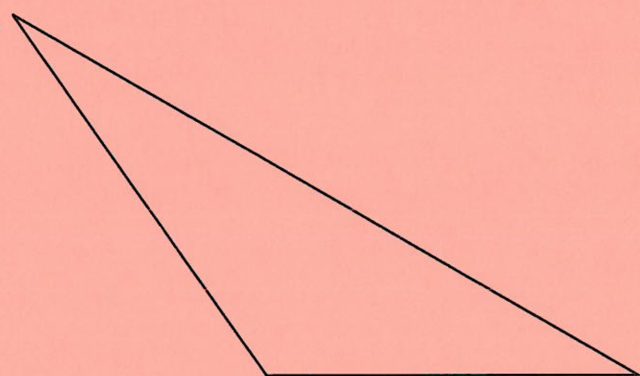
1.



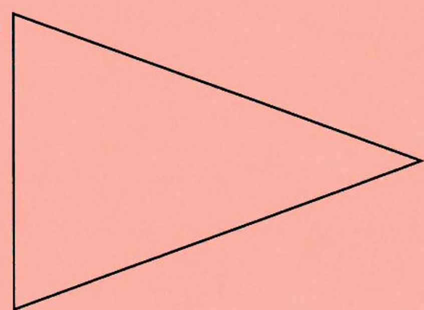
2.



3.



4.



Day 2 Challenge II: Drawing Triangles

What is the smallest number of side and angle measurements that will tell you how to draw an exact copy of any given triangle?

Launch:

In this Problem, students use this knowledge to look at the conditions that produce unique triangles. Suppose you want to text a friend to give directions for drawing an exact copy of the figure.

Questions

• What is the shortest message to do the job? After some class discussion of approaches, guide your students by suggesting that they look for some general guidelines they could use for writing their messages about triangles with minimum text and maximum information.

Explore:

As you circulate, help students to connect their number-line work to the work of developing an algorithm for rewriting mixed numbers as improper fractions and improper fraction as mixed numbers.

Questions

• How did you decide on the placement of -54 on the number line?

• How did you conclude that -54 is to the right of -1?

You want to follow up this line of reasoning with questions about greater and less than as they pertain to negative numbers.

Summarize:

The fundamental question to be answered in this Problem is Question C.

Side 1 side 2 side 3---Triangle?? Can a triangle be built with any three side lengths. Students should have a strong sense that the answer.

Questions

• What minimum information about a triangle allows you to draw exactly one triangle?

They should know from their work, that given three side lengths that form a triangle, there is only one triangle (SSS condition). When students are ready, discuss other conditions that produce unique triangles: ASA and SAS.

Minnesota Standard:

____ **9.3.3.7 Use properties of polygons—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results. *For example*: Recognize that a rectangle is a special case of a trapezoid. *Another example*: Give a concise and clear definition of a kite.**

What combinations of side lengths can be used to make a quadrilateral? How many different shapes are possible for any such combination of side lengths?

Launch:

You might choose to launch this Problem by simply suggesting that students can experiment with polystrips to see how construction and properties of quadrilaterals are similar to and different from triangles.

Questions

- Can you make a quadrilateral using any four lengths for the sides?
- If so, is the shape unique?

Students conduct an experiment to gather data to help answer the questions above. Explain to students that keeping an accurate record of their data is very important because it allows them to recreate examples as evidence of what they discovered.

Explore/Share:

Encourage them to consider different arrangements of side lengths and to record differences in the shapes that may occur. Also, encourage students to make sketches of their quadrilaterals.

Questions:

- In another class, a group said they thought they could make more than one quadrilateral with the lengths 6, 8, 10, and 12. They said, "When we put the 10 between the 6 and the 8, the quadrilateral is different from the one we get when we put the 10 between the 8 and the 12."
- What do you think about this group's idea? You may want to provide extra polystrips so that students can keep various versions of quadrilaterals with the same side lengths to compare. This way they can check to see if the quadrilaterals are the same or different before disassembling them.

Summarize/Share:

Each Question of this Problem focuses on an important property of polygons; so it will be important to review student answers. Two powerful strategies that focus on different aspects of what determines a quadrilateral are the following:

- Put the set of lengths together in different orders. (This technique highlights the role of side lengths in determining a shape.)
- Build a quadrilateral from polystrips. Alter its shape by pressing on the sides or vertices of the quadrilateral. A quadrilateral with any given side lengths can form an infinite number of different

quadrilaterals. (This technique highlights the role of angles in determining a shape and the lack of rigidity for quadrilaterals.) If four side lengths make a quadrilateral, the shape is not unique.

Day 4 will be on Parallel lines and Transversals
Not in this lesson plan group

Day 5 The Quadrilateral Game

How are squares, rhombuses, rectangles, and trapezoids similar?
How are they different?

Launch:
Displaying the quilt designs. Bring in my mom's homemade Quilts. So talk about Quilting store in Remer. Ask the students what makes the quilts so appealing to the eye. You could then ask students how the displayed quilts seem to have each type of symmetry.
Describe the Quadrilateral Game.
Display the rules
Quadrilateral Game Rules

• Near the center of the geoboard, put the rubber band around a square measuring one unit on each side.
• Team A rolls the number cubes one at a time to locate an entry in the game grid on the next page. The first number locates the row and the second number locates the column. Team A reads the description in this location. Team A then looks at the quadrilateral already on the game board, and forms a new quadrilateral to match the description. They move as few corners of the already existing quadrilateral as possible. Team A receives one point for each corner moved.
• Next, Team B rolls the number cubes and locates the corresponding description on the grid. They make a quadrilateral fitting the new description by moving as few corners of the existing quadrilateral as possible. Team B receives one point for each corner moved.
• Play continues until each team has had five turns. The team with the lowest score at the end is the winner.
Next:
. Then divide the class into two teams. To demonstrate how to play. Once you are sure students understand how to play the game, pair students and have two pairs play each other.

Explore:
Let the groups explore symmetry in the figures of the Shapes Set.
Questions
• If I put a pin in the center of an equilateral triangle to hold it in place, how can I turn it so that it looks the same as its starting position?
• Can I turn the triangle some more to find another Rotation Symmetry?
Help students to see that you can turn the triangle another third of a full turn, or 120.

As they play the game tell students to keep track of strategies and difficult situations.

1. When did you receive 0 points during a turn? Why didn't you need to move any corners on those turns?
2. Write two new descriptions of quadrilaterals that you could use in the game grid?
3. Make your own game board with description of for a triangle Game.

Row 6	A quadrilateral that is a square	**Add 1 point to your score and skip your turn**	A rectangle that is not a square	A quadrilateral with two obtuse angles	A quadrilateral with exactly one pair of parallel sides	A quadrilateral with one pair of opposite side lengths equal
Row 5	**Subtract 2 points from your score and skip your turn**	A quadrilateral that is not a rectangle	A quadrilateral with two pairs of consecutive angles that are equal	A quadrilateral with all four angles the same size	A quadrilateral with four lines of symmetry	A quadrilateral that is a rectangle
Row 4	A quadrilateral with no reflection or rotation symmetry	A quadrilateral with four right angles	**Skip a turn**	A quadrilateral with exactly one pair of consecutive side lengths that are equal	A quadrilateral with exactly one right angle	A quadrilateral with two 45° angles
Row 3	A quadrilateral with no angles equal	A quadrilateral with one pair of equal opposite angles	A quadrilateral with exactly one pair of opposite angles that are equal	**Add 2 points to your score and skip your turn**	A quadrilateral with no sides parallel	A quadrilateral with exactly two right angles
Row 2	A quadrilateral with both pairs of adjacent side lengths equal	A quadrilateral with two pairs of equal opposite angles	A quadrilateral with a diagonal that divides it into two identical shapes	A quadrilateral that is a rhombus	A quadrilateral with 180° rotation symmetry	**Subtract 1 point from your score and skip your turn**
Row 1	A quadrilateral with one diagonal that is a line of symmetry	A quadrilateral with no side lengths equal	A quadrilateral with exactly one angle greater than 180°	A parallelogram that is not a rectangle	**Add 3 points to your score and skip your turn**	A quadrilateral with two pairs of opposite side lengths equal
	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6

Summarize/Share:

Focus the discussion on only one example of each type of symmetry from the triangles and quadrilaterals in the Shapes Set. To make sure that students have the right idea about both Reflection and Rotation Symmetry, include a nonrectangular parallelogram in your questioning. Since this problem is combining the knowledge developed in prior Problems from this Unit, a simple summary question like the following might be sufficient.

Questions

- What did you find particularly interesting as you played the game? • Can someone give me an example of when they received zero points during a turn?
- What about one, two, three, or four points?
- What does it mean when a figure on the geoboard already matches the new description?
- Which sets of descriptions on the grid are equivalent? • Which situations were particularly challenging or interesting?

4.2 Building Quadrilaterals

TEKS / TAKS 6(11)C Select an appropriate problem-solving strategy including looking for a pattern.
6(13)A Make conjectures from patterns.

You need four side lengths to make a quadrilateral.

Will any four side lengths work?

Can you make more than one quadrilateral from four side lengths?

Problem 4.2 Building Quadrilaterals

active math online
For: Linkage Strips Activity
Visit: PHSchool.com
Web Code: amd-3402

A. **1.** Use polystrips to build quadrilaterals with each of the following sets of numbers as side lengths. Try to build two or more different quadrilaterals using the same set of side lengths.

6, 10, 15, 15	3, 5, 10, 20
8, 8, 10, 10	12, 20, 6, 9

Sketch and label your results to share with your classmates. Record any observations you make.

2. Choose your own sets of four numbers and try to build quadrilaterals with those numbers as side lengths.

B. Use your observations from Question A.

1. Is it possible to make a quadrilateral using any four side lengths? If not, how can you tell whether you can make a quadrilateral from four side lengths?

2. Can you make two or more different quadrilaterals from the same four side lengths?

3. What combinations of side lengths are needed to build rectangles? Squares? Parallelograms?

C. **1.** Use four polystrips to build a quadrilateral. Press on the sides or corners of your quadrilateral. What happens?

2. Use another polystrip to add a diagonal connecting a pair of opposite vertices. Now, press on the sides or corners of the quadrilateral. What happens? Explain.

D. **1.** Describe the similarities and differences between what you learned about building triangles in Problem 4.1 and building quadrilaterals in this problem.

2. Explain why triangles are used in building structures more often than quadrilaterals.

ACE **Homework starts on page 76.**

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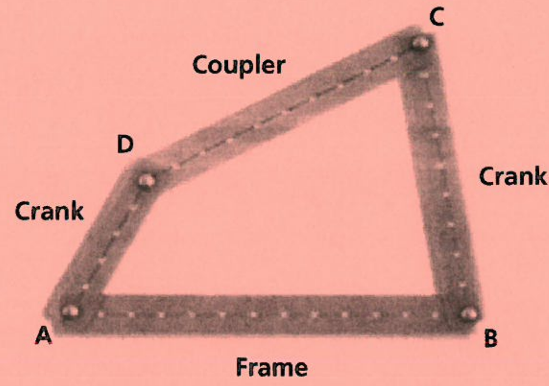
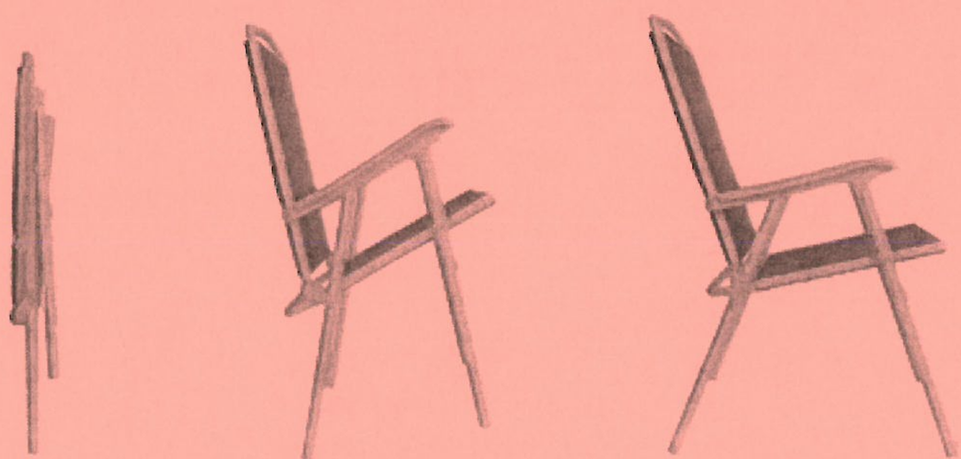
Did You Know?

Mechanical engineers use the fact that quadrilaterals are not rigid to design *linkages*. Below is an example of a quadrilateral linkage.



One of the sides is fixed. It is the *frame*. The two sides attached to the frame are the *cranks.* One of the cranks is the driver and the other the follower. The fourth side is called the *coupler*. Quadrilateral linkages are used in windshield wipers, automobile jacks, reclining lawn chairs, and handcars.



In 1883, the German mathematician Franz Grashof suggested an interesting principle for quadrilateral linkages: If the sum of the lengths of the shortest and longest sides is less than or equal to the sum of the lengths of the remaining two sides, then the shortest side can rotate 360°.

Name .... Date .... Class ....

Labsheet 3.3 Building Quadrilaterals

Side 1	Side 2	Side 3	Side 4	Quadrilateral? (yes or no)	Sketch

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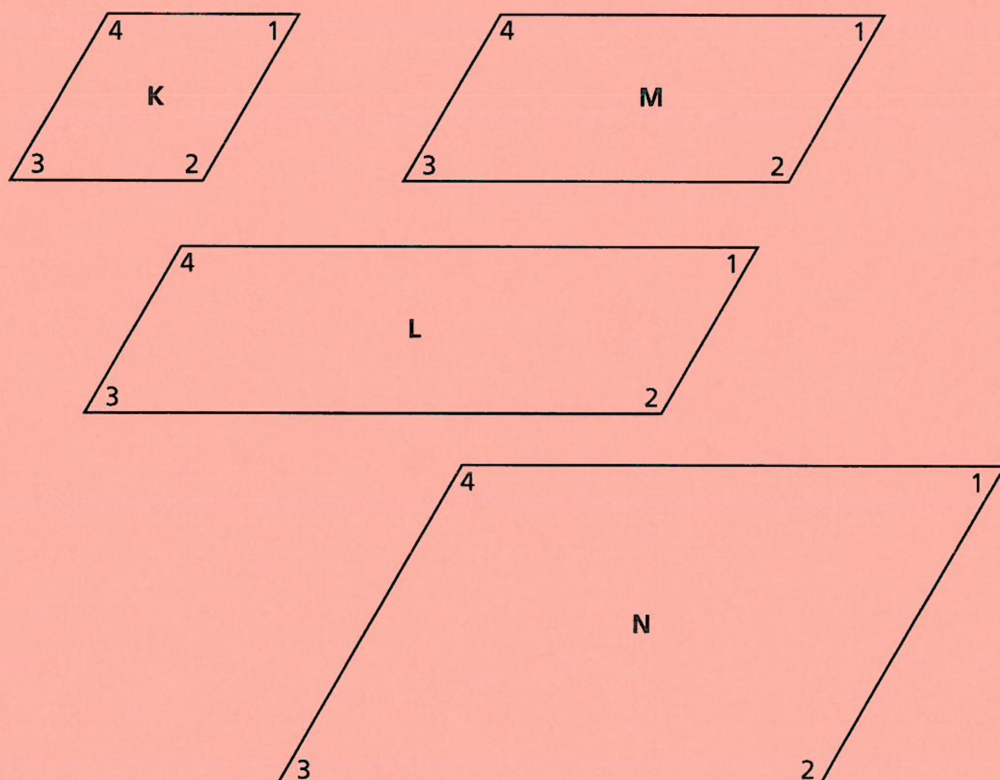
Name .... Date .... Class ....

Labsheet 3.4A

Parallelograms

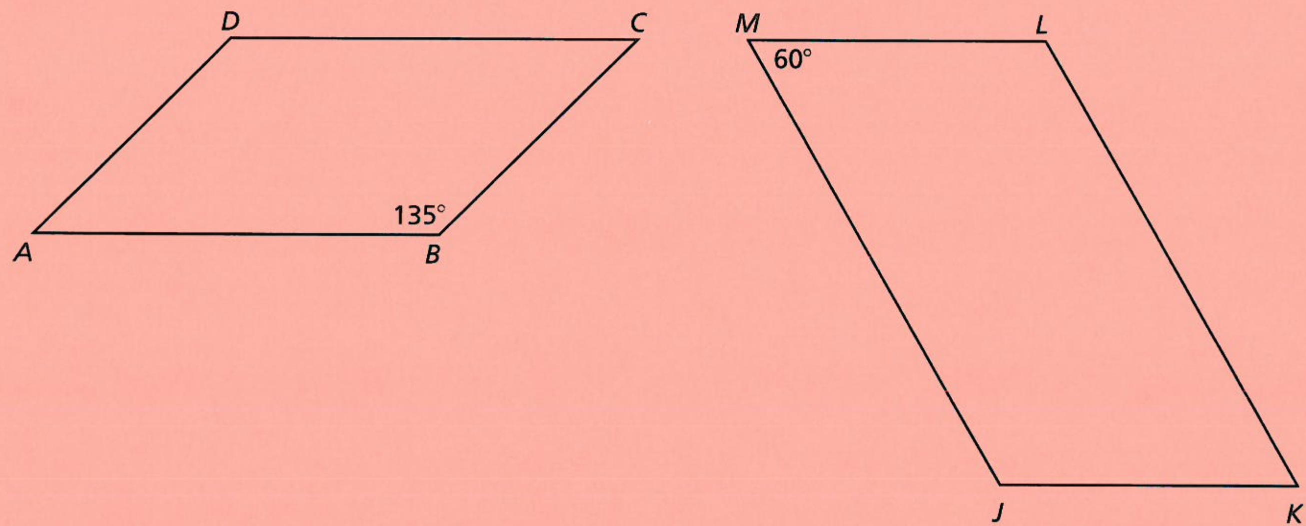


Name .... Date .... Class ....

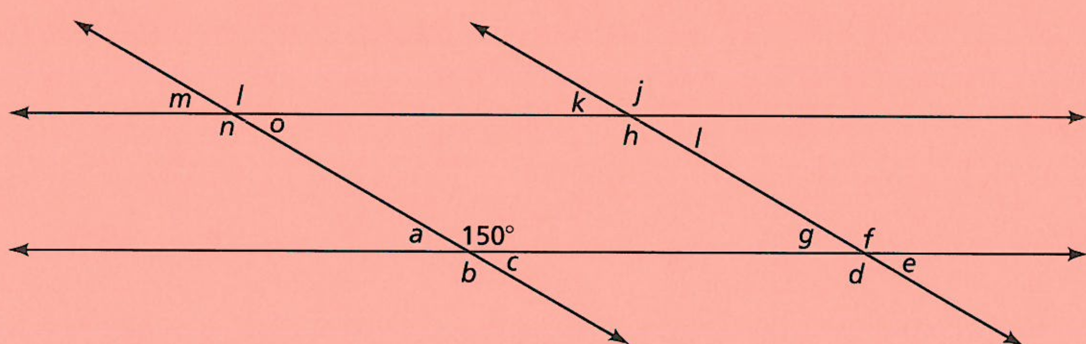
Labsheet 3.4B

Questions A–E

What are the measures of the angles in parallelograms *ABCD* and *JKLM* below?



Find the measures of all labeled angles in this diagram. Be prepared to justify each answer.



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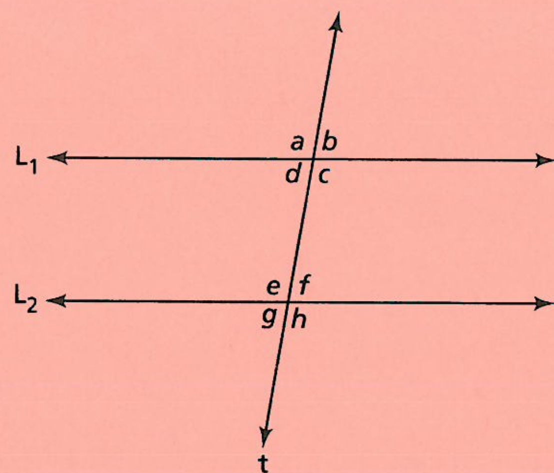
Name .... Date .... Class ....

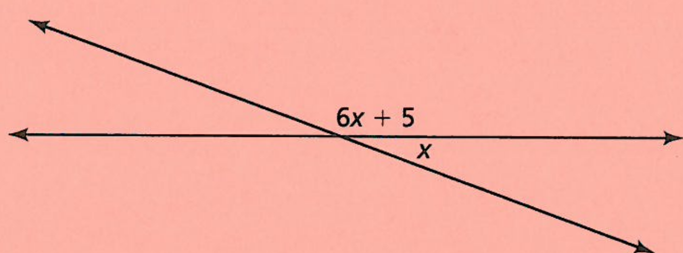
Labsheet 3.4B

Questions A–E

Suppose the measure of angle e is 80°. What are the measures of the other labeled angles?



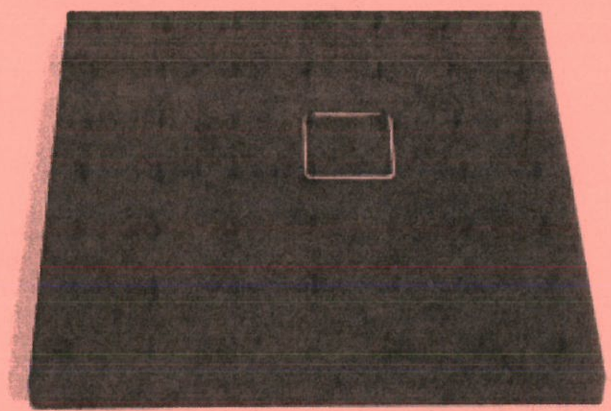
Find the value of x and the size of each angle in this figure.



4.3 The Quadrilateral Game

TEKS / TAKS **6(11)C** Select an appropriate problem-solving strategy including looking for a pattern.
6(13)A Make conjectures from patterns.

The Quadrilateral Game will help you explore the properties of quadrilaterals. The game is played by two teams. To play, you need two number cubes, a game grid, a geoboard, and a rubber band.



Quadrilateral Game Rules

- Near the center of the geoboard, put the rubber band around a square measuring one unit on each side.
- Team A rolls the number cubes one at a time to locate an entry in the game grid on the next page. The first number locates the row and the second number locates the column. Team A reads the description in this location. Team A then looks at the quadrilateral already on the game board, and forms a new quadrilateral to match the description. They move as few corners of the already existing quadrilateral as possible. Team A receives one point for each corner moved.
- Next, Team B rolls the number cubes and locates the corresponding description on the grid. They make a quadrilateral fitting the new description by moving as few corners of the existing quadrilateral as possible. Team B receives one point for each corner moved.
- Play continues until each team has had five turns. The team with the lowest score at the end is the winner.

Problem 4.3 Properties of Quadrilaterals

A. Play the Quadrilateral Game. Keep a record of interesting strategies and difficult situations. Make notes about when you do not receive a point during a turn. Why did you not need to move any corners on those turns?

B. Write two new descriptions of quadrilaterals that you could include in the game grid.

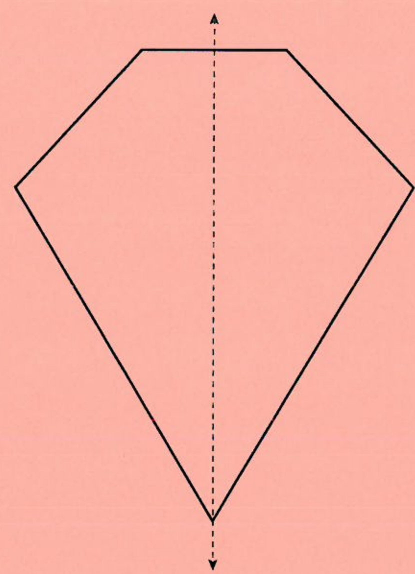
active math online
For: Quadrilateral Game Activity
Visit: PHSchool.com
Web Code: amd-3403

ACE **Homework starts on page 76.**

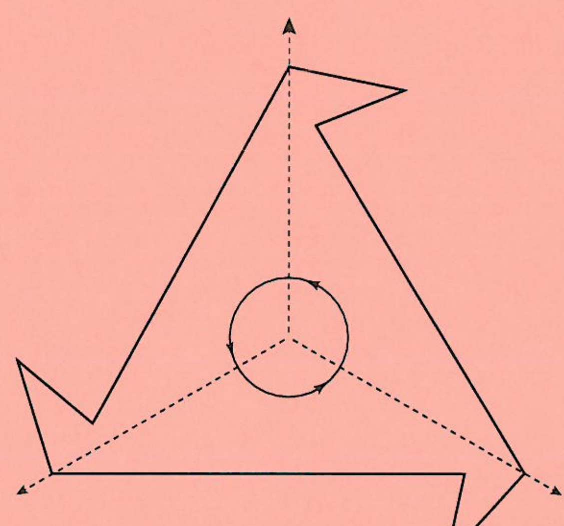
Quadrilateral Game Grid

Row 6	A quadrilateral that is a square	**Add 1 point to your score and skip your turn**	A rectangle that is not a square	A quadrilateral with two obtuse angles	A quadrilateral with exactly one pair of parallel sides	A quadrilateral with one pair of opposite side lengths equal
Row 5	**Subtract 2 points from your score and skip your turn**	A quadrilateral that is not a rectangle	A quadrilateral with two pairs of consecutive angles that are equal	A quadrilateral with all four angles the same size	A quadrilateral with four lines of symmetry	A quadrilateral that is a rectangle
Row 4	A quadrilateral with no reflection or rotation symmetry	A quadrilateral with four right angles	**Skip a turn**	A quadrilateral with exactly one pair of consecutive side lengths that are equal	A quadrilateral with exactly one right angle	A quadrilateral with two 45° angles
Row 3	A quadrilateral with no angles equal	A quadrilateral with one pair of equal opposite angles	A quadrilateral with exactly one pair of opposite angles that are equal	**Add 2 points to your score and skip your turn**	A quadrilateral with no sides parallel	A quadrilateral with exactly two right angles
Row 2	A quadrilateral with both pairs of adjacent side lengths equal	A quadrilateral with two pairs of equal opposite angles	A quadrilateral with a diagonal that divides it into two identical shapes	A quadrilateral that is a rhombus	A quadrilateral with 180° rotation symmetry	**Subtract 1 point from your score and skip your turn**
Row 1	A quadrilateral with one diagonal that is a line of symmetry	A quadrilateral with no side lengths equal	A quadrilateral with exactly one angle greater than 180°	A parallelogram that is not a rectangle	**Add 3 points to your score and skip your turn**	A quadrilateral with two pairs of opposite side lengths equal
	Column 1	**Column 2**	**Column 3**	**Column 4**	**Column 5**	**Column 6**

Reflection Symmetry



Rotation Symmetry



Quadrilateral Game Rules

- Near the center of the geoboard, put the rubber band around a square measuring one unit on each side.
- Team A rolls the number cubes one at a time to locate an entry in the game grid on the next page. The first number locates the row and the second number locates the column.
- Team A reads the description in that location. Then they look at the quadrilateral already on the game board, and form a new quadrilateral to match the description. The challenge for Team A is to move as few corners as possible to make the new quadrilateral.
- For each corner moved, Team A receives one point.
- Next, Team B rolls the number cubes and locates the corresponding description on the grid. They make a quadrilateral matching the new description by moving as few of the corners as possible. Team B receives one point for each corner moved.
- Play continues until each team has had five turns. The team with the lowest score at the end is the winner.

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Name .... Date .... Class ....

Labsheet 3.5

Quadrilateral Game Grid

Row 6	A quadrilateral that is a square	**Add 1 point to your score and skip your turn**	A rectangle that is not a square	A quadrilateral with two obtuse angles	A quadrilateral with exactly one pair of parallel sides	A quadrilateral with one pair of opposite side lengths equal
Row 5	**Subtract 2 points from your score and skip your turn**	A quadrilateral that is not a rectangle	A quadrilateral with two pairs of consecutive angles that are equal	A quadrilateral with all four angles the same size	A quadrilateral with four lines of symmetry	A quadrilateral that is a rectangle
Row 4	A quadrilateral with no reflection or rotation symmetry	A quadrilateral with four right angles	**Skip a turn**	A quadrilateral with exactly one pair of consecutive side lengths that are equal	A quadrilateral with exactly one right angle	A quadrilateral with two 45° angles
Row 3	A quadrilateral with no angles equal	A quadrilateral with one pair of equal opposite angles	A quadrilateral with exactly one pair of opposite angles that are equal	**Add 2 points to your score and skip your turn**	A quadrilateral with no sides parallel	A quadrilateral with exactly two right angles
Row 2	A quadrilateral with both pairs of adjacent side lengths equal	A quadrilateral with two pairs of equal opposite angles	A quadrilateral with a diagonal that divides it into two identical shapes	A quadrilateral that is a rhombus	A quadrilateral with 180° rotation symmetry	**Subtract 1 point from your score and skip your turn**
Row 1	A quadrilateral with one diagonal that is a line of symmetry	A quadrilateral with no side lengths equal	A quadrilateral with exactly one angle greater than 180°	A parallelogram that is not a rectangle	**Add 3 points to your score and skip your turn**	A quadrilateral with two pairs of opposite side lengths equal
	Column 1	**Column 2**	**Column 3**	**Column 4**	**Column 5**	**Column 6**

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Applications Connections Extensions ACE

Applications

Follow these directions for Exercises 1–4.

- If possible, build a triangle with the given set of side lengths. Sketch your triangle.
- Tell whether your triangle is the only one that is possible. Explain.
- If a triangle is not possible, explain why.

1. Side lengths of 5, 5, and 3

2. Side lengths of 8, 8, and 8

3. Side lengths of 7, 8, and 15

4. Side lengths of 5, 6, and 10

5. Which set(s) of side lengths from Exercises 1–4 can make each of the following shapes?

a. an equilateral triangle

b. an isosceles triangle

c. a scalene triangle

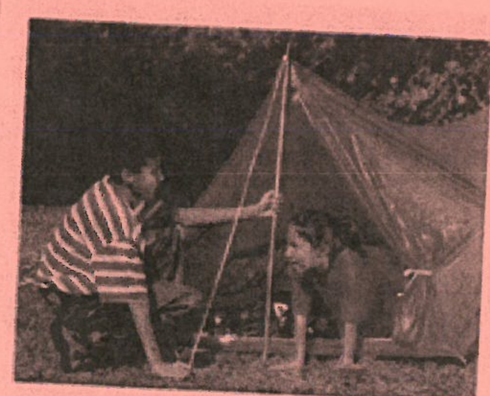
d. a triangle with at least two angles of the same measure

For Exercises 6 and 7, draw the polygons described to help you answer the questions.

6. What must be true of the side lengths in order to build a triangle with three angles measuring 60°? What kind of triangle is this?

7. What must be true of the side lengths in order to build a triangle with only two angles the same size? What kind of triangle is this?

8. Giraldo and Maria are building a tent. They have two 3-foot poles. In addition, they have a 5-foot pole, a 6-foot pole, and a 7-foot pole. They want to make a triangular-shaped doorframe for the tent using both 3-foot poles and one of the other poles. Which of the other poles could be used to form the base of the door?



Follow these directions for Exercises 9–12.

- If possible, build a quadrilateral with the given set of side lengths. Sketch your quadrilateral.
- Tell whether your quadrilateral is the only one that is possible. Explain.
- If a quadrilateral is not possible, explain why.

9. Side lengths of 5, 5, 8, and 8

10. Side lengths of 5, 5, 6, and 14

11. Side lengths of 8, 8, 8, and 8

12. Side lengths of 4, 3, 5, and 14

13. Which set(s) of side lengths from Exercises 9–12 can make each of the following shapes?

a. a square

b. a quadrilateral with all angles the same size

c. a parallelogram

d. a quadrilateral that is not a parallelogram

14. A quadrilateral with four equal sides is called a **rhombus.** Which set(s) of side lengths from Exercises 9–12 can make a rhombus?

15. A quadrilateral with at least one pair of parallel sides is called a **trapezoid.** Which set(s) of side lengths from Exercises 9–12 can make a trapezoid?

For Exercises 16 and 17, draw the polygons described to help you answer the questions.

16. What must be true of the side lengths of a polygon to build a square?

17. What must be true of the side lengths of a polygon to build a rectangle that is not a square?

18. Li Mei builds a quadrilateral with sides that are each five inches long. To help stabilize the quadrilateral, she wants to insert a ten-inch diagonal. Is this possible? Explain.

Homework Help Online
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19. You are playing the Quadrilateral Game. The shape currently on the geoboard is a square. Your team rolls the number cubes and gets the description "A parallelogram that is not a rectangle." What is the minimum number of vertices your team needs to move to form a shape meeting this description?

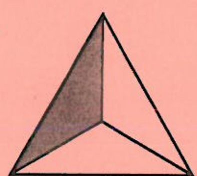
Applications

20. You are playing the Quadrilateral Game. The shape currently on the geoboard is a non-rectangular parallelogram. Your team rolls the number cubes and gets the description "A quadrilateral with two obtuse angles." What is the minimum number of vertices your team needs to move to create a shape meeting this description?

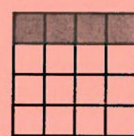
Connections

21. Multiple Choice Which one of the following shaded regions is *not* a representation of $\frac{4}{12}$?

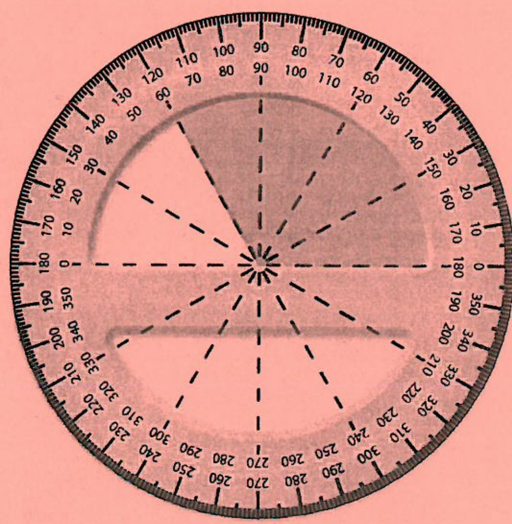
A.



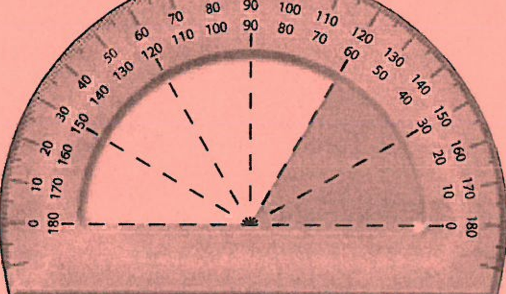
B.



C.

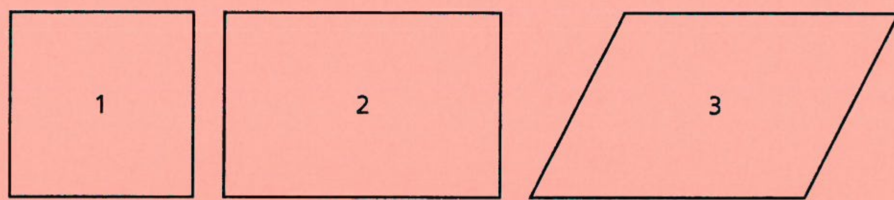


D.

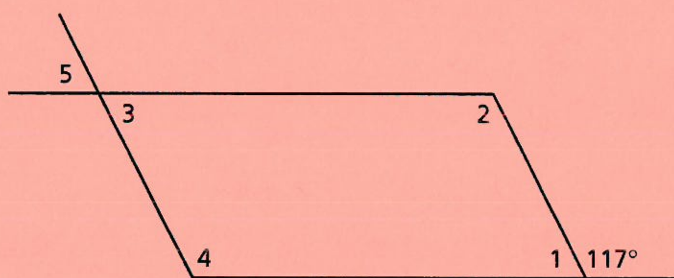


22. **a.** How are all three quadrilaterals below alike?

b. How does each quadrilateral differ from the other two?



23. In this parallelogram, find the measure of each numbered angle.



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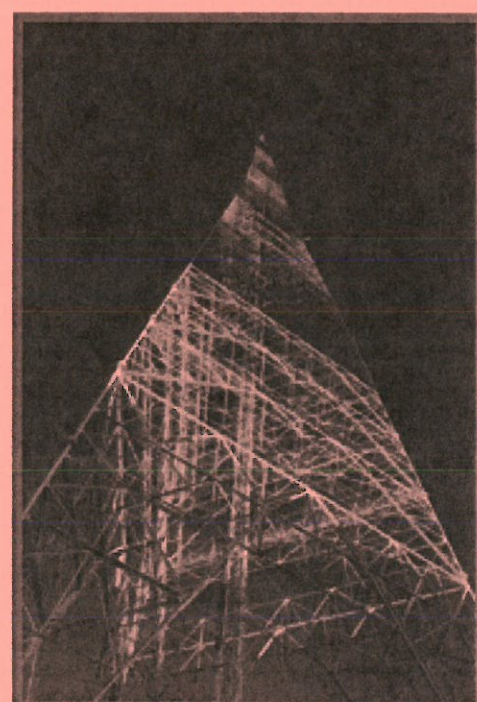
24. Think about your polystrip experiments with triangles and quadrilaterals. What explanations can you now give for the common use of triangular shapes in structures like bridges and antenna towers for radio and television?



Multiple Choice **For Questions 25–28, choose the symmetry or symmetries of each shape.**

25. rhombus (four equal sides)

F. rotational **G.** reflectional

H. both F and G **J.** none

26. regular pentagon

A. rotational **B.** reflectional

C. both A and B **D.** none

27. square

F. rotational **G.** reflectional

H. both F and G **J.** none

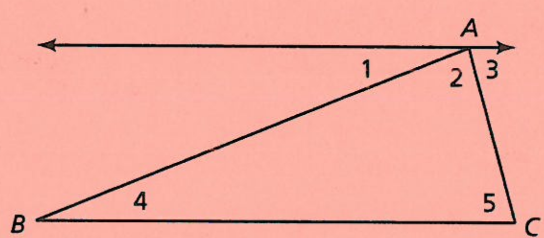
28. a parallelogram that is not a rhombus or a rectangle

A. rotational **B.** reflectional

C. both A and B **D.** none

Extensions

29. In the triangle, a line has been drawn through vertex *A*, parallel to line segment *BC* of the triangle.



a. What is the sum of the measures of angles 1, 2, and 3?

b. Explain why angle 1 has the same measure as angle 4 and why angle 3 has the same measure as angle 5.

c. How can you use the results of parts (a) and (b) to show that the angle sum of a triangle is 180°?

30. In parts (a)–(c), explore pentagons by using polystrips or by making sketches.

a. If you choose five numbers as side lengths, can you always build a pentagon? Explain.

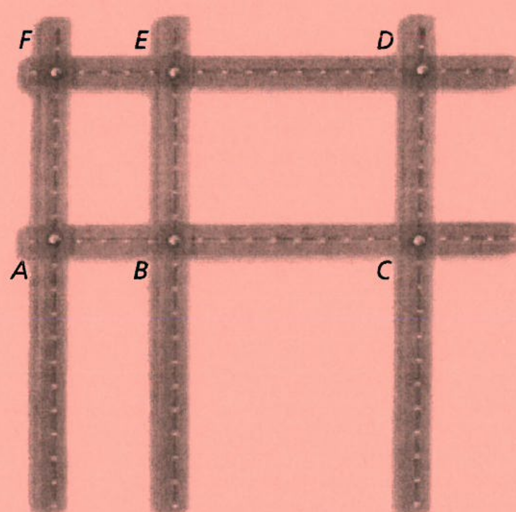
b. Can you make two or more different pentagons from the same side lengths?

31. Refer to the *Did You Know?* after Problem 4.2.

a. Make a model that illustrates Grashof's principle using polystrips or paper fasteners and cardboard strips. Describe the motion of your model.

b. How can your model be used to make a stirring mechanism? A windshield wiper?

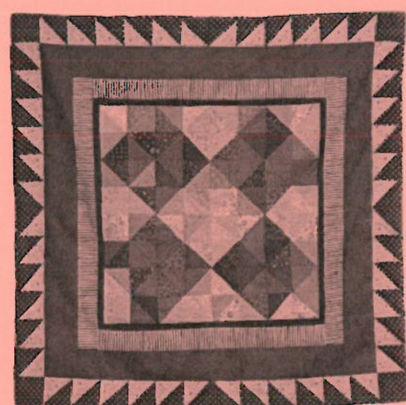
32. Build the figure below from polystrips. Note that the vertical sides are all the same length, the distance from B to C equals the distance from E to D, and the distance from B to C is twice the distance from A to B.



a. Experiment with holding various strips fixed and moving the other strips. In each case, tell which strips you held fixed, and describe the motion of the other strips.

b. Fix a strip between points F and B and then try to move strip CD. What happens? Explain why this occurs.

Connections Extensions



What symmetries can you find in the quilts pictured above?

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Best Route

Launch:
Project the Road Trip Overhead (or map from the website). Present the problem: they are traveling salespeople on a road trip for their company. They will start in Deer River, then travel through Pine River, Hill City, Cass Lake, and Federal Dam(not necessarily in that order). Ask students, "How could we plan this trip so that our traveling time is the shortest possible route?" (Some ideas might include: drive to the closest city you haven't visited, pick the lowest values from the table and use those to form a route, or calculate all possible routes and pick the shortest one.) Use computer to fill out table of how far it is from one city to each of the other cities.

Explore:
1) Group students in small groups of 2-3 Students will calculate the shortest route by choosing to drive to the closest city they haven't visited. Compare answers as a class before moving on.

Summerize:

Students will use weighted vertex-edge graphs to calculate the shortest distance by finding all of the shortest distances and connecting as many cities as possible before using longer distances.
Discuss the vertex-edge graphs and student answers before moving on. Ask students:

- "Why can't you have 3 routes going to and from the same city?"
- "Why can't you have a route that creates a "mini-tour" of cities (a closed circuit)?

Ask students these questions:

- "In the 4 city example, how many choices did we have for the next city as we left city A?"
- After that city, how many choices for the next?
- After that, how many choices remained?
- For n cities, we find the formula = $(n-1)!/2$

As a class, decide on which method gave the "best" answer for this problem.

Review the algorithms used in this problem: Nearest Neighbor, (or "Greedy"). Ask students to briefly describe each.
Discuss with students: besides best route salesman career, when else is this type of problem-solving (for finding the shortest route) useful? Which algorithm did they think was most useful and why? What factors might change their choice?

	Pine River	Hill City	Federal Dam	Cass Lake	Deer River
Pine River	0				
Hill City		0			
Federal Dam			0		
Cass Lake				0	
Deer River					0

Lesson: Interior and Exterior Angles in Polygons

Objective: Students will be able to draw and classify polygons based on the number of sides. Students will be able to measure the interior and exterior angles and develop an understanding of properties of the interior and angles of polygons.

Standards: 9.3.1.5 Make reasonable estimates and judgments about the accuracy of values resulting from calculations involving measurements.

Launch: We will start with a discussion about the different classifications of polygons (including by sides, regular or irregular, concave or convex). They will be familiar with triangles and perhaps quadrilaterals, but the instructor will challenge them to find the sum of the interior and exterior angles angles in a 20-gon and a method for finding the same angles in any polygon (n-gon), regardless of the number of sides.

Explore: Students will be working in pairs with a ruler and protractor and draw each of the different polygons, one polygon per side of paper (sides from 4-8, all must be convex and irregular). They will measure all interior and exterior angles and record the results on the drawing. Then they will make a chart with the sum of the interior angles and the sum of the exterior angles for the polygons.

Share: Students will share their results by posting their drawings around the room for all to see. They will be asked to identify a pattern in the results and try to use that pattern to find the sum of the interior and exterior angles in a 20-gon and in any polygon (n-gon). Each of the groups will be asked to share their results with the class.

Summarize: The instructor will facilitate a class discussion in order to bring everyone to a consensus, that being that all polygons have exterior angles whose sum is 360 degrees and any n-gon has the sum of the interior angles being (n-1)180 degrees.

Interior and Exterior Angles in Polygons

With a ruler and protractor draw the following irregular convex polygons. Measure and record the angles on the of the polygons and complete the following chart.

Polygon	Sum of interior angles	Sum of exterior angles
Quadrilateral		
Pentagon		
Hexagon		
Heptagon		
Octagon		

1. Try to identify a pattern to help you determine what each answer should be for the interior angles. What is the pattern?

2. Try to identify the pattern for the exterior angles. What is the pattern?

3. Find the sum of the interior angles of a 20-gon.

4. Find the sum of the exterior angles of a 20-gon.

5. How would you find the sum of the interior angles of an *n*-gon? Where *n* represents any number of sides.

Name________________________

Discrete Math Post Test

1. Find the sum of the interior angles in a convex 18-gon.

2. Find the sum of the exterior angles in a convex 10-gon.

3. Find the measure of one interior angle of a regular 30-gon.

4. Find the measure of one exterior angle of a regular Octagon.

5. A piece of wood is cut into a triangle with two sides that measure 19 cm. The angles of the triangle measure 60°, 60°, and 60°. Describe the triangle according to its sides and angles.

   A. The triangle is an equilateral acute triangle.

   B. The triangle is a scalene obtuse triangle.

   C. The triangle is a scalene right triangle.

   D. The triangle is a scalene acute triangle.

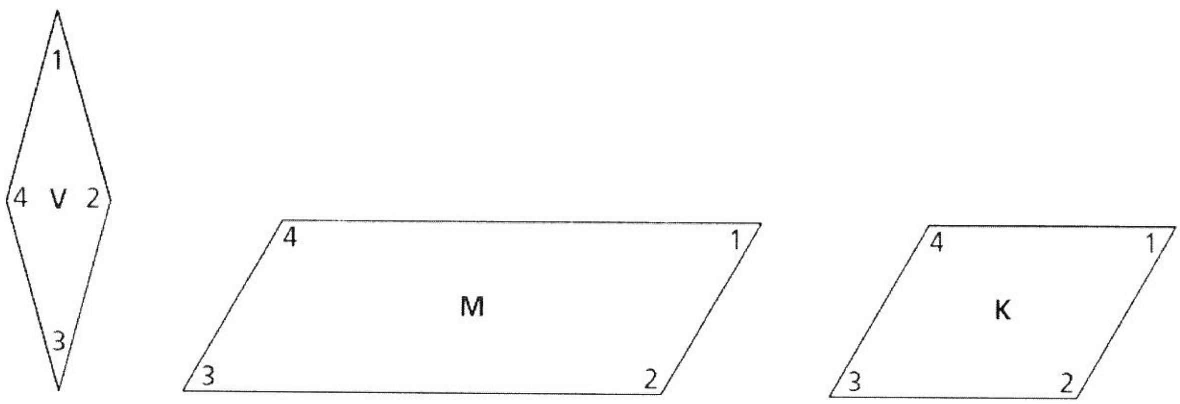
6. Which statement about quadrilaterals is true?
   A. All trapezoids are parallelograms.
   B. All parallelograms are squares.
   C. All rhombuses are parallelograms.
   D. All rectangles are squares.

7. Draw and label the polygon with the following properties.
   <ABC = 90, <BCA = 45, and side BC = 1 in.

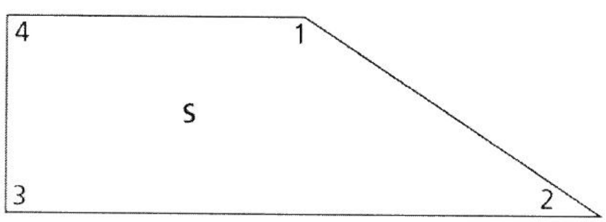
8. The new computer desk for your room has enough shelf space to have three reference books. Your parents purchased a set of four reference books for you. How many different ways can you place three of the reference books on the shelf of your new desk?

9.

Autumn places the following shapes into a group.



The shape below does not belong in the group.



Give one reason Autumn put shapes V, M, and K in a group.

10. Your family wants to take a road trip to visit some of your relatives. Your relatives live in St. Cloud, Duluth, and Shakopee.
Use the Table below to help answer the following questions.

	Detroit Lakes	St. Cloud	Duluth	Shakopee
Detroit Lakes	X	136	207	216
St. Cloud	136	X	144	78
Duluth	207	144	X	178
Shakopee	216	78	178	X

A.) Draw a vertex-edge graph connecting all of the cities together. Make the vertices represent the towns and the edges represent the distances between the towns.

B.) Use the "nearest neighbor" algorithm to find the shortest route.